AQA Qualifications

AQA Level 2 Certificate

FURTHER MATHEMATICS

Level 2 (8360)

Assessment Guidance

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Contents

1 Number

1.1 Number

Assessment Guidance

Candidates should be able to

- understand and use the correct hierarchy of operations
- understand and use decimals, fractions and percentages
- understand and use ratio and proportion
- understand and use numbers in index form and standard form
- understand rounding and give answers to an appropriate degree of accuracy.

Notes

Knowledge, use and understanding of the numerical processes in the Key Stage 4 Programme of Study will be expected.

Candidates will not be expected to use symbols for real numbers, natural numbers or integers.

The symbol $\sqrt{\ }$ will be taken to mean the positive square root of a number, although candidates should be aware that a positive number has two distinct square roots.

Questions will only be set in the context of other references within the specification.

Examples

1 $x : y = 2 : 3$

(a) Write y in terms of x.

(b) Work out the ratio $2x : x + 2y$

Give your answer in its simplest form

2 a% of $2b$ = b% of $5c$

Work out c as a percentage of a.

3 A ship travels 15 km due South from *A* to *B*.

It then travels 18 km due East from *B* to *C*.

Work out the distance *AC*.

Give your answer to an appropriate degree of accuracy.

1.2 Manipulation of surds, including rationalising the denominator

Assessment Guidance

Candidates should be able to

- simplify expressions by manipulating surds
- expand brackets which contain surds
- rationalise the denominator, including denominators in the form $a\sqrt{b} + c\sqrt{d}$ where a, b, c and d are integers
- understand the concept of using surds to give an exact answer.

Notes

Questions may be set in the context of other references within the specification.

Examples

1 Work out the exact value of $\sqrt{3} \times \sqrt{6} \times \sqrt{8}$

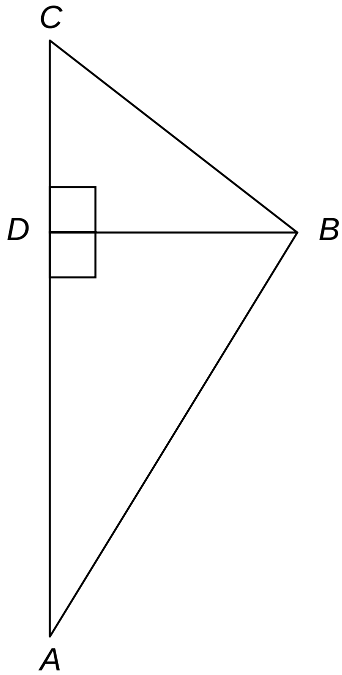
2 Solve $\sqrt{98} - \sqrt{18} = \sqrt{50} - \sqrt{x}$

3 Express $(4 + \sqrt{3})(1 - 2\sqrt{3})$ in the form $a + b\sqrt{3}$

4 In the diagram $AB = 7$ cm, $BC = 2$ cm and $CD = \sqrt{3}$ cm

Show that $AD = 4\,CD$

Not drawn accurately



2 Algebra

2.1 The basic processes of algebra

Assessment Guidance

Candidates should be able to

- understand and use commutative, associative and distributive laws
- understand and use the hierarchy of operations
- recall and apply knowledge of the basic processes of algebra, extending to more complex expressions, equations, formulae and identities.

Notes

Knowledge and understanding of the algebraic processes in the Key Stage 4 Programme of Study will be expected. Questions which require these processes may be set as a single problem or with structure given.

Examples

1 Simplify fully $\dfrac{6a + a \times 4a}{2a}$

2 Solve $3(x + 2) - 4 = 2(5 - x)$

3 (a) Factorise $25x^2 - 16$

(b) Factorise $15x^2 - 7x - 4$

(c) Simplify $\dfrac{25x^2 - 16}{15x^2 - 7x - 4}$

2.2 Definition of a function

Assessment Guidance

Candidates should be able to

- understand that a function is a relation between two sets of values
- understand and use function notation, for example $f(x)$
- substitute values into a function, knowing that, for example $f(2)$ is the value of the function when $x = 2$
- solve equations that use function notation.

Notes

Questions requiring explicit knowledge of composite functions and inverse functions will not be set.

Examples

1 Given that $f(x) = 4x - 5$ work out

(a) $f(-6)$

(b) $f(0.5)$

2 $f(x) = 3x + 2$

Solve $f(x) = 0$

3 $f(x) = x^2 + 3x - 6$

(a) Write down an expression for $f(2x)$

(b) Solve $f(2x) = 0$
Give your answer to 2 decimal places.

2.3 Domain and range of a function

Assessment Guidance

Candidates should be able to

- define the domain of a function
- work out the range of a function
- express a domain in a variety of forms, for example $x > 2$, for all x except $x = 0$, for all real values
- express a range in a variety of forms, for example $f(x) \leq 0$, for all $f(x)$ except $f(x) = 1$.

Notes

Candidates will not be expected to use $\forall$ or the symbols for real numbers, natural numbers or integers.

Examples

1 Given that $f(x) = x^2$ and that $x > 3$, state the range of $f(x)$.

2 Given that $f(x) = 3x + 4$ and that $f(x) < 0$, state the largest possible integer value of x.

3 $f(x) = 2x + 3$ and $x > 6$

(a) Write down an expression for $f(3x)$

(b) State the domain and range of the function $f(3x)$

2.4 Expanding brackets and collecting like terms

Assessment Guidance

Candidates should be able to

- expand two or more brackets
- simplify expressions by collecting like terms.

Notes

Brackets may contain polynomials of any degree. Terms may include negative or fractional powers. Candidates should understand the phrases 'descending order' and 'ascending order'. Knowledge of Pascal's triangle and binomial expansions are not required.

Examples

1 Expand and simplify $(x^4 + 5x^3 - 4x^2 + 7x)(x - 4)$

2 Expand and simplify $(2x + 5)^3$

3 Expand and simplify $(y^2 + 5y - 2)(y^2 + 7) - 3(y^3 - 2y)$

4 Expand $x^{\frac{1}{2}}(x + 5)$

5 Expand and simplify $(x + \frac{1}{x})(x - \frac{1}{x})$

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2.5 Factorising

Assessment Guidance

Candidates should be able to

- factorise by taking out common factors from expressions
- factorise expressions given in the form of a quadratic
- factorise a difference of two squares.

Notes

Candidates will be expected to give answers in simplest form.

Examples

1 Factorise $4x^2 - 25$

2 Factorise $12x^2 + xy - y^2$

3 Factorise fully $8x^3 - 72x$

4 Factorise $(3x + 7)^2 - (2x - 3)^2$

2.6 Manipulation of rational expressions: Use of + – × ÷ for algebraic fractions with denominators being numeric, linear or quadratic Factorising

Assessment Guidance

Candidates should be able to

- use a combination of the skills required for sections 2.1, 2.4 and 2.5 in order to manipulate and simplify rational algebraic expressions.

Notes

Questions may involve more than one variable. Questions may involve polynomials of degree greater than 2 but these will either factorise to a polynomial of degree no greater than 2 multiplied by a common factor or be in the form of a quadratic.

Examples

1 Simplify fully $\dfrac{6x^2 + 7x - 3}{16x^2 - 1} \div \dfrac{2x^3 + 3x^2}{4x + 1}$

2 Simplify fully $\dfrac{5x}{(x + 1)(x - 4)} - \dfrac{4}{(x - 4)}$

3 Simplify fully $\dfrac{x^4 - 5x^3 + 6x^2}{x^4 - 13x^2 + 36}$

2.7 Use and manipulation of formulae and expressions

Assessment Guidance

Candidates should be able to

- change the subject of a formula, where the subject appears on one or both sides of the formula
- manipulate formulae and expressions
- show how one side of an identity can be manipulated to obtain the other side of the identity.

Notes

Candidates will be expected to show all steps of working using a systematic approach. Candidates will be expected to give answers in simplest form.

Examples

1 Make x the subject of $5y = 4 - 3x$

2 Make y the subject of $3y + 7 = 8xy - 1$

3 Make z the subject of $\frac{1}{z} + \frac{x}{2y} = x$

4 Show that $\frac{1}{x+3} + \frac{4}{x-5}$ simplifies to $\frac{5x+7}{x^2 - 2x - 15}$

2.8 Use of the factor theorem for integer values of the variable, including cubics

Assessment Guidance

Candidates should be able to

- understand and use the factor theorem to factorise polynomials up to and including cubics
- show that $x - a$ is a factor of the function $f(x)$ by checking that $f(a) = 0$
- solve equations up to and including cubics, where at least one of the roots is an integer.

Notes

Cubic equations will have three real solutions unless there are repeated roots.

When the factor theorem is needed to factorise a given cubic, the coefficient of x^3 will always be 1.

Examples

1 Factorise $x^3 + x^2 - 5x + 3$

2 Show that $x - 3$ is a factor of $x^3 - 4x^2 + x + 6$

3 Solve $x^3 - 4x^2 + x + 6 = 0$

2.9 Completing the square

Assessment Guidance

Candidates should be able to

- complete the square for any quadratic function of the form $ax^2 + bx + c$ where a, b and c are integers
- solve quadratic equations by completing the square
- equate coefficients to obtain unknown values.

Notes

The identity symbol should be known and understood.

Examples

1 Work out the values of a, b and c such that $4x^2 + 8x - 1 \equiv a(x + b)^2 + c$

2 Work out the values of a, b and c such that $3x^2 + bx + 4 \equiv a(x + 2)^2 + c$

3 Work out the values of p and q such that $6 - 4x - x^2 \equiv p - (x + q)^2$

4 (a) Work out the values of a, b and c such that $3x^2 + 4x - 1 \equiv a(x + b)^2 + c$

(b) Hence or otherwise, solve the equation $3x^2 + 4x - 1 = 0$, giving your answers to 3 significant figures.

2.10 Sketching of functions

Sketch graphs of linear and quadratic functions

Assessment Guidance

Candidates should be able to

- draw or sketch graphs of linear and quadratic functions with up to 3 domains
- label points of intersection of graphs with the axes
- understand that graphs should only be drawn within the given domain
- identify any symmetries on a quadratic graph and from this determine the coordinates of the turning point.

Notes

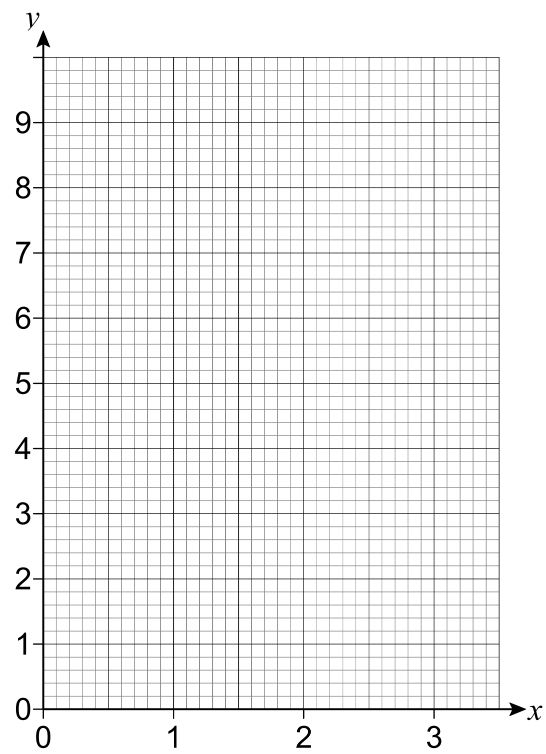
Axes and/or labels may or may not be given in the question. Graphs may or may not be continuous.
Graphs of cubic functions are covered in section 4.6.

Examples

1 A function f(x) is defined as

$$f(x) = \begin{cases} 1 & 0 \le x < 1 \\ 4 & 1 \le x < 2 \\ x^2 & 2 \le x \le 3 \end{cases}$$

Draw the graph of $y = $ f(x) on the grid.

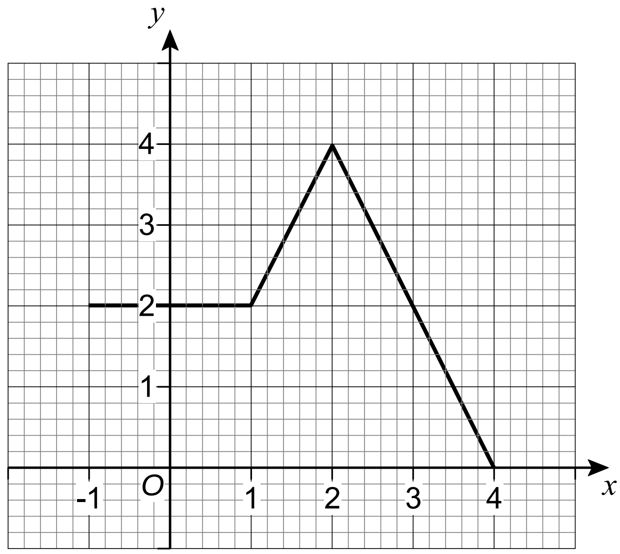


2 (a) Sketch the graph of $y = 2x^2 - x - 3$

Label clearly any points of intersection with the axes.

(b) Work out the equation of the line of symmetry of the graph of $y = 2x^2 - x - 3$ for all real values of x.

3 Here is a graph of $y = f(x)$.



Define $f(x)$, stating clearly the domain for each part.

2.11 Solution of linear and quadratic equations

Assessment Guidance

Candidates should be able to

- solve linear equations
- solve quadratic equations by factorisation, by graph, by completing the square or by formula.

Notes

Questions may require the setting up of linear or quadratic equations from a variety of contexts. Questions may require the solutions to be interpreted, for example give the positive answer as it represents a length.

Examples

1 The length of a rectangle is $(2x + 1)$ cm.

The width of the rectangle is $(x + 7)$ cm.

The area of the rectangle is 444 cm^2.

Set up an equation in x.

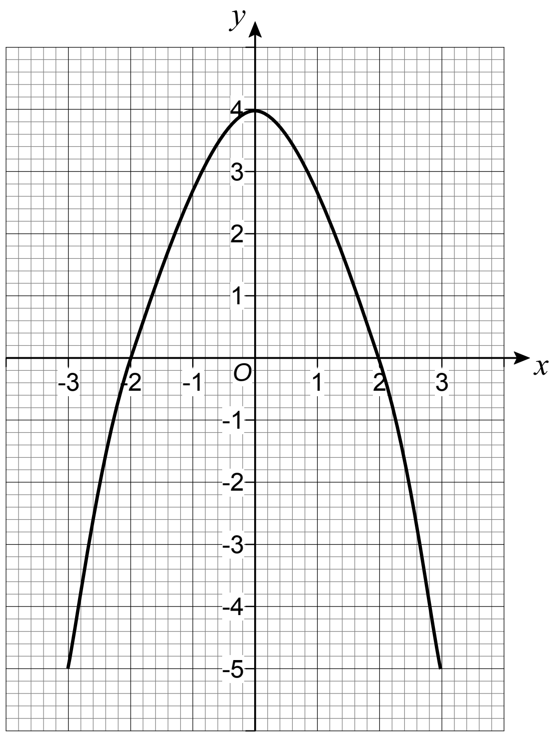
Solve your equation and hence work out the perimeter of the rectangle.

2 The length of the base of a triangle is double the length of the perpendicular height.

The area of the triangle is 150 cm^2.

Work out the length of the base of the triangle to two significant figures.

3 The graph of $y = 4 - x^2$ is shown.



By drawing a suitable linear graph on the grid work out approximate solutions to $x^2 + x - 4 = 0$

Give solutions to one decimal place.

2.12 Algebraic and graphical solution of simultaneous equations in two unknowns where the equations could be both linear or one linear and one second order

Assessment Guidance

Candidates should be able to

- solve two linear simultaneous equations using any valid method
- solve simultaneous equations where one is linear and one is second order using substitution or any other valid method.

Notes

Questions may require the setting up of linear or quadratic equations from a variety of contexts.

Examples

1 Solve the simultaneous equations $3x + 5y = 34$ and $5x + 3y = 30$

2 Solve the simultaneous equations $x^2 + y^2 = 45$ and $x + 2y = 9$

3 Solve the simultaneous equations $xy = 6$ and $5x - 2y = 7$

2.13 Solution of linear and quadratic inequalities

Assessment Guidance

Candidates should be able to

- solve linear inequalities
- solve quadratic inequalities.

Notes

Questions may require factorisation of quadratics. Questions may require solution by completing the square for a quadratic expression or using a graphical method.

Questions may require integer values that satisfy the inequalities.

Questions may ask for the smallest or largest integer value that satisfies the inequality.

Examples

1 Solve $8(x - 7) < 3(x + 2)$

2 Solve $5x^2 + 13x - 6 \geq 0$

3 Work out the integer values that satisfy the inequality $4x^2 + 7x - 15 < 0$

2.14 Index laws, including fractional and negative indices

Assessment Guidance

Candidates should be able to

- simplify expressions involving fractional and negative indices which may be written in a variety of forms
- solve equations involving expressions involving fractional and negative indices
- understand that, for example $x^{\frac{1}{n}}$ is equivalent to the nth root of x
- understand that, for example x^{-n} is equivalent to $\dfrac{1}{x^n}$.

Notes

Questions may be set in index notation or using roots and reciprocals.

Examples

1 Solve $x^{\frac{3}{2}} = 8$

2 Simplify $\sqrt{x^{\frac{7}{2}} \times x^{-\frac{3}{2}}}$

3 Solve $x^3 = 3\sqrt{3}$

Give your answer as a power of 3.

2.15 Algebraic proof

Assessment Guidance

Candidates should be able to

- show that an expression can be manipulated into another given form
- prove given conditions for algebraic expressions.

Notes

Candidates will be expected to give elegant solutions to problems following standard mathematical conventions. The presentation of algebraic proofs should be rigorous.

Examples

1 Prove algebraically that the sum of three consecutive integers is always a multiple of 3.

2 (a) Show that $\frac{2x+5}{3} + \frac{3x-1}{2}$ simplifies to $\frac{13x+7}{6}$

(b) Hence solve $\frac{2x+5}{3} + \frac{3x-1}{2} = 8$

3 Prove that $(2x+3)^2 - 3x(x+4)$ is always positive.

2.16 nth terms of linear and quadratic sequences. Limiting value of a sequence as $n \to \infty$

Assessment Guidance

Candidates should be able to

- write down the value of the nth term of a sequence for any given value of n
- work out a formula for the nth term of a sequence, which may contain linear or quadratic parts
- work out the limiting value for a given sequence or for a given nth term as n approaches infinity.

Notes

Sequences may use a combination of linear and quadratic sequences, for example when the sequence involves fractions such as $\dfrac{n^2 + 1}{3n - 2}$

Examples

1 The nth term of a sequence is $\dfrac{7 - n}{n^2 + 1}$

(a) Which term in the sequence is the first one that has a negative value?

(b) Work out the value of this term.

2 Work out the limiting value of $\dfrac{n}{5n + 1}$ as $n \to \infty$

3 Work out the formula for the nth term of the quadratic sequence 5 11 19 29 …..

3 Coordinate Geometry (2 dimensions only)

3.1 Know and use the definition of a gradient

Assessment Guidance

Candidates should be able to

- work out the gradient of a line given two points on the line
- select two points on a given line to work out the gradient
- use the gradient of a line and a known point on the line to work out the co-ordinates of a different point on the line.

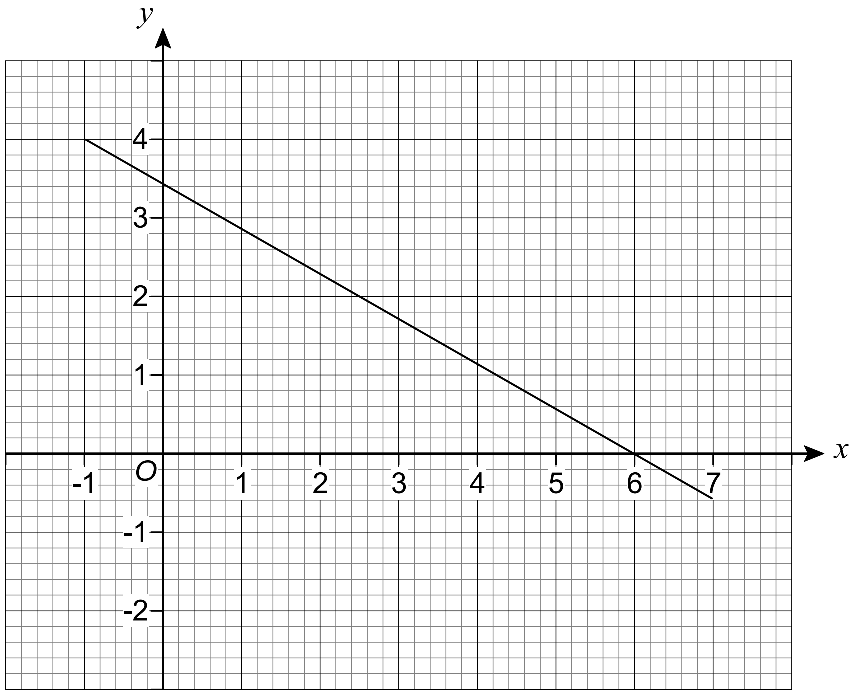
Notes

Candidates will be expected to give exact values for gradients in their simplest form.

Examples

1 Work out the gradient of the line that passes through (1, –2) and (–1, 5)

2 Work out the gradient of the line shown.



3 A line has gradient 3 and passes through the points (6, –1) and (a, $2a$).

Work out the value of a.

3.2 Know the relationship between the gradients of parallel and perpendicular lines

Assessment Guidance

Candidates should be able to

- work out the gradients of lines that are parallel and perpendicular to a given line
- show that two lines are parallel or perpendicular using gradients.

Notes

Candidates may be required to use angle properties listed in section 6.1 and section 3.7.

Solution by accurate drawing will not be acceptable.

Examples

1 Here are the equations of two straight lines

$y = 2 - 3x \qquad 6x + 2y = 1$

Show that the lines are parallel.

2 Work out the gradient of a line that is perpendicular to the line $2x + 5y = 6$

3 *A* is (2, 3), *B* is (5, 8), *C* is (7, 6) and *D* is (1, –4).

Show that *ABCD* is a trapezium.

3.3 Use Pythagoras' theorem to calculate the distance between two points

Assessment Guidance

Candidates should be able to

- recall the formula or use a sketch diagram to obtain the appropriate lengths of sides.

Notes

Candidates may be required to give an exact answer.

Candidates may be required to use mensuration formulae listed in section 6.1.

Solution by accurate drawing will not be acceptable.

Examples

1 Work out the distance *PQ* where *P* is (–2, 3) and *Q* is (4, 1)

Give your answer in the form $a\sqrt{b}$ where a and b are integers.

2 *A* is (4, 1), *B* is (5, 3) and *C* is (8, –1).

(a) Show that *AB* is perpendicular to *AC*.

(b) Work out the area of triangle *ABC*.

3.4 Use ratio to find the coordinates of a point on a line given the coordinates of two other points

Assessment Guidance

Candidates should be able to

- use the formula for the coordinates of the midpoint
- use a given ratio to work out coordinates of a point given two other points.

Notes

A formula can be used when the ratio is not 1 : 1 but any appropriate method will be acceptable.

Solution by accurate drawing will not be acceptable.

Examples

1 *C* is (6, 2).
The midpoint of *CD* is (8, –4).
Work out the coordinates of *D*.

2 *A* is (–4, 15) and *B* is (10, –6).
Point *C* is such that *AC* : *CB* is 4 : 3
Work out the coordinates of *C*.

3.5 The equation of a straight line in the forms: $y = mx + c$ and $y - y_1 = m(x - x_1)$

Assessment Guidance

Candidates should be able to

- work out the gradient and the intercepts with the axes of a given equation or graph
- work out the equation of a line using the gradient and a known point on the line
- work out the equation of a line using two known points on the line
- give equations in a particular form when instructed to do so
- work out coordinates of the point of intersection of two lines.

Notes

Any correct form will be acceptable if a particular form is not asked for.

Examples

1 A straight line has equation $3x + 2y = 8$

(a) Work out the gradient of the line.

(b) Work out the coordinates of where the line intersects the x-axis.

(c) Work out the coordinates of where the line intersects the y-axis.

2 Work out the equation of the straight line that passes through (–2, 3) and (1, 5).

Give your answer in the form $y = mx + c$.

3 Work out the coordinates of the point of intersection of the lines $y = 2x - 1$ and $y = 7 - 3x$

3.6 Draw a straight line from given information

Assessment Guidance

Candidates should be able to

- draw a straight line using a given gradient and a given point on the line
- draw a straight line using two given points on the line.

Notes

Candidates may need to make their own table of values.

Examples

1 Draw the line $y = 3x - 5$ for values of x from -3 to 3.

2 Draw the line that has gradient $\frac{1}{2}$ and intersects the y-axis at 5.

Use x values from 0 to 8.

3.7 Understand that the equation of a circle, centre (0, 0), radius r is $x^2 + y^2 = r^2$

Assessment Guidance

Candidates should be able to

- recognise the equation of a circle, centre (0, 0), radius r
- write down the equation of a circle given centre (0, 0) and radius
- work out coordinates of points of intersection of a given circle and a given straight line.

Notes

Candidates are expected to know the definitions of common words associated with circles.

Candidates may need to apply circle geometry facts.

Examples

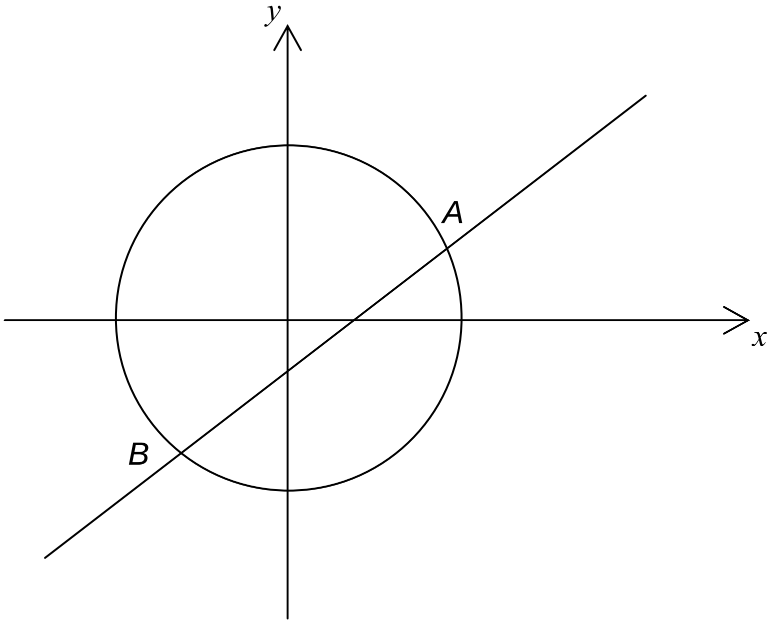
1 The equation of a circle is $x^2 + y^2 = 2$

(a) Write down the coordinates of the centre of the circle.

(b) Write down the exact value of the radius of the circle.

2 Write down the equation of a circle, centre (0, 0) and radius 5.

3 The circle $x^2 + y^2 = 10$ and the line $y = x - 2$ intersect at points A and B as shown.



Work out the length of the chord AB.

3.8 Understand that $(x-a)^2+(y-b)^2=r^2$ is the equation of a circle with centre (a, b) and radius r

Assessment Guidance

Candidates should be able to

- recognise the equation of a circle, centre (0, 0), radius r
- write down the equation of a circle given centre (0, 0) and radius
- work out coordinates of points of intersection of a given circle and a given straight line.

Notes

Candidates are expected to know the definitions of common words associated with circles.

Candidates may need to apply circle geometry facts.

Candidates are expected to write equations in the form $(x-a)^2+(y-b)^2=r^2$ and will not be expected to expand the brackets but will be expected to evaluate r^2.

Translations should be described using a vector.

Examples

1 The equation of a circle is $(x-3)^2+(y+2)^2=16$

(a) Write down the coordinates of the centre of the circle.

(b) Write down the radius of the circle.

(c) Describe the transformation that maps this circle to the circle $x^2+y^2=16$

2 Write down the equation of a circle, centre (–1, 4) and radius $\sqrt{5}$.

3 *P* is (–2, –3) and *Q* is (4, 5).
PQ is a diameter of a circle.

Work out the equation of the circle.

4 Calculus

4.1 Know that the gradient function $\frac{dy}{dx}$ gives the gradient of the curve and measures the rate of change of y with respect to x

Assessment Guidance

Candidates should be able to

- understand and use the notation $\frac{dy}{dx}$
- understand the concept of the gradient of a curve
- understand the concept of a rate of change
- use the skills of 4.3 to work out gradients of curves and rates of change.

Notes

The notation δx, δy and $\frac{\delta y}{\delta x}$ will not be used in questions.

Examples

1 A curve has the gradient function $\frac{dy}{dx} = 4x^2 + 3$

(a) What is the gradient of the curve when $x = 2$?

(b) Work out the values of x for which the rate of change of y with respect to x is 4.

2 Work out the gradient of the curve $y = 4x^2 - 2x + 3$ at the point $(\frac{1}{2}, 3)$.

3 Given that $y = 3x - x^2$

Work out the coordinates of the point at which the gradient of the curve is 5.

4 Given that $y = x(4 + x)$

Work out the rate of change of y with respect to x when $x = 3$.

4.2 Know that the gradient of a function is the gradient of the tangent at that point

Assessment Guidance

Candidates should be able to

- understand the concept of the gradient of a curve
- state the gradient of a curve at a point given the gradient or equation of the tangent at that point
- state the gradient of the tangent at a point given the gradient of the curve at that point
- use the skills of 4.1 and 4.3 to work out gradients of curves and tangents.

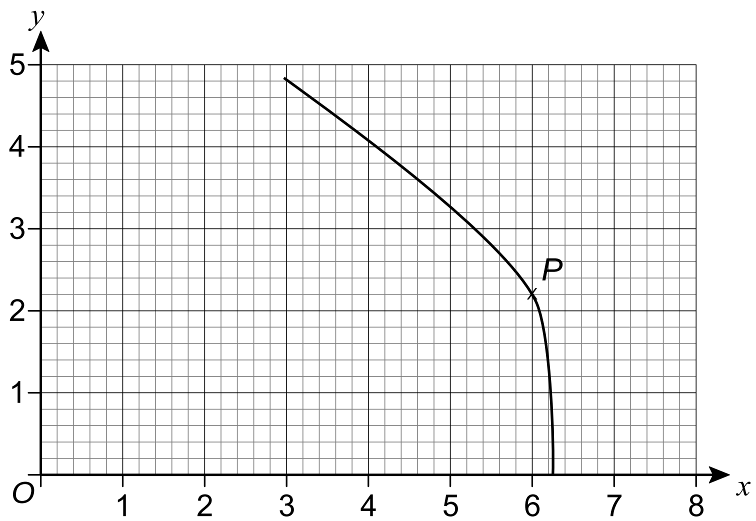
Notes

The notation δx, δy and $\dfrac{\delta y}{\delta x}$ will not be used in questions.

Examples

1 The diagram shows the point P on the curve $y = \mathrm{f}(x)$

By drawing a suitable line estimate the gradient of the curve at P.



2 Q is the point (1, 3) and O is the origin.

(a) Show that Q lies on the curve $y = 2x^2 - x + 2$

(b) Show that the line through O and Q is a tangent to the curve $y = 2x^2 - x + 2$

3 The tangent at the point R on the curve $y = 5x^2 + 6x + 1$ is parallel to the line $y = 7 - 4x$.

Work out the coordinates of R.

4.3 Differentiation of kx^n where n is a positive integer or 0, and the sum of such functions

Assessment Guidance

Candidates should be able to

- find $\frac{dy}{dx}$, where $y = kx^n$ where k is a constant and n is a positive integer or 0
- simplify expressions before differentiating if necessary.

Notes

Other rules of differentiation (eg the product rule) are not expected.

Questions will not be set which require differentiation from first principles.

Examples

1 Work out $\frac{dy}{dx}$

(a) $y = 5x^3 + 6x - 3$ (b) $y = x^2(x + 2)$ (c) $y = (x + 8)(x - 4)$

(d) $y = \frac{x^2 + 7}{4}$ (e) $y = (3x + 2)^2$

4.4 The equation of a tangent and normal at any point of a curve

Assessment Guidance

Candidates should be able to

- use the skills of 4.2, 4.3 and 3.5 to work out the equation of a tangent to a curve
- use the skills of 4.2, 4.3, 3.2 and 3.5 to work out the equation of a normal to a curve.

Notes

Questions may require answers to be given in the form $y = mx + c$

Examples

1 Work out the equation of the tangent to the curve $y = x^2 - 3x + 5$ at the point (1, 3)

2 Work out the equation of the normal to the curve $y = x^3 + x - 2$ at the point where $x = -1$
Give your answer in the form $y = mx + c$

3 (a) Work out the equation of the normal to the curve $y = 6 - x^2$ at the point P (1, 5)

(b) The normal intersects the curve again at Q.
Work out the coordinates of Q.

4.5 Use of differentiation to find stationary points on a curve: maxima, minima and points of inflection

Assessment Guidance

Candidates should be able to

- understand that stationary points are points at which the gradient is zero
- use the skills of 4.3 to work out stationary points on a curve
- understand the meaning of increasing and decreasing functions
- understand the meaning of maximum points, minimum points and points of inflection
- prove whether a stationary point is a maximum, minimum or point of inflection.

Notes

It is expected that in questions which ask for a maximum, minimum or inflection point, candidates will prove the nature of the point they have found.

Questions will not use the term 'turning points' and the distinction between turning points and stationary points is not expected.

Knowledge of the second derivative is not expected but may be used.

Applications of maxima and minima in real life situations will **not** be set.

Questions may require candidates to work out the values of x for which a function is increasing or decreasing.

Examples

1 Prove that the curve $y = x^3 + 3x^2 + 3x - 2$ has only one stationary point.

Show that the stationary point is a point of inflection.

2 Show that the curve $y = 4x - x^4$ has only 1 stationary point.

Determine the nature of this point.

3 Show that $y = \frac{1}{3}x^3 - 3x^2 + 10x - 2$ is an increasing function for all values of x.

4 For what values of x is $y = x^3 - 3x^2 + 5$ a decreasing function?

4.5 Sketch a curve with known stationary points

Assessment Guidance

Candidates should be able to

- draw a sketch graph of a curve having used the skills of 4.5 to work out the stationary points.

Notes

Candidates will be expected to sketch curves which have more than one stationary point.

Candidates will be expected to label the stationary points and where appropriate the intercepts with the x and y axes.

Candidates should take care in sketching a stationary point which is a point of inflection.

Examples

1 (a) Solve the equation $x^3 + 3x^2 = 0$

(b) Work out the coordinates of the stationary points on the curve $y = x^3 + 3x^2$

(c) Sketch of the curve $y = x^3 + 3x^2$

2 (a) Show that the curve $y = 4x - x^4$ has only 1 stationary point.

(b) Sketch the curve $y = 4x - x^4$

5 Matrix Transformations

5.1 Multiplication of matrices

Assessment Guidance

Candidates should be able to

- multiply a 2 × 2 matrix by a 2 × 1 matrix
- multiply a 2 × 2 matrix by a 2 × 2 matrix
- multiply 2 × 2 and 2 × 1 matrices by a scalar
- understand that, in general, matrix multiplication is not commutative
- understand that matrix multiplication is associative.

Notes

Questions will involve finding the image (or object) point under a given transformation matrix.

Elements of matrices could be numerical or algebraic.

Questions may be set using index notation.

Candidates will be expected to be familiar with the process of equating elements of equal matrices.

Candidates will **not** be expected to find the inverse of a 2 × 2 matrix

Examples

1 $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

Work out

(a) 3**A** (b) **AB** (c) 2**BA** (d) **BC** (e) $\mathbf{A}^2$

2 Matrix $\mathbf{A} = \begin{pmatrix} 5 & 3 \\ 2 & -1 \end{pmatrix}$

Work out the image of the point (1, –2) using the transformation represented by **A**.

3 The image of a point P (x, y) is (9, 1) using the transformation represented by $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$

Work out x and y.

4 Given $\begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ a \end{pmatrix} = \begin{pmatrix} 9 + b \\ 3b \end{pmatrix}$

Work out a and b.

5.2 Identify the matrix, I

Assessment Guidance

Candidates should be able to

- understand that **AI** = **IA** = **A**

Notes

The identity matrix, **I**, will be used in the context of matrix transformations.

Candidates will **not** be expected to find the inverse of a 2 × 2 matrix.

Examples

1 $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Show that $\mathbf{A}^2 = \mathbf{I}$

2 $\mathbf{A} = \begin{pmatrix} 0 & 4 \\ 2 & -1 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 3 & 12 \\ 6 & 0 \end{pmatrix}$

Show that $\mathbf{AB} = 24\mathbf{I}$

5.3 Transformations of the unit square in the $x - y$ plane

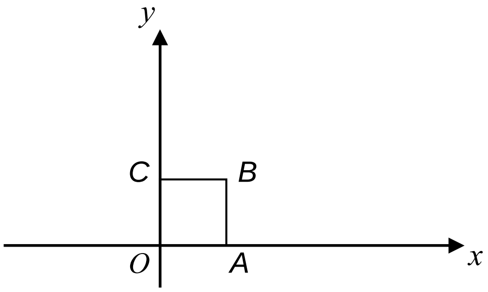
Assessment Guidance

Candidates should be able to

- work out the image of any vertex of the unit square given the matrix operator
- work out or recall the matrix operator for a given transformation

Notes

Transformations will be limited to transforming the unit square.



Transformations of the unit square will be restricted to rotations through 90°, 180° and 270° about the origin, reflections in the axes and the lines $y = x$ and $y = -x$, and enlargements centred on the origin.

For enlargements the term scale factor should be known. Both positive and negative scale factors will be used.

Candidates will be expected to understand the notation A' to mean the image of a point A under a transformation.

Candidates will not be expected to understand the phrase ‘invariant point’.

The knowledge and use of unit vectors **i** and **j** is not required.

Examples

1 A (1, 0) and C (0, 1) are opposite vertices of the unit square $OABC$.

The square is mapped to $OA'B'C'$ under transformation matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Work out the coordinates of A', B', and C'.

2 The matrix **M** represents a reflection in the line $y = -x$.

Work out **M**.

3 The matrix $\mathbf{M} = 3\mathbf{I}$

Describe geometrically the transformation represented by **M**.

5.4 Combination of Translations

Assessment Guidance

Candidates should be able to

- understand that the matrix product **PQ** represents the transformation with matrix **Q** followed by the transformation with matrix **P**
- use the skills of 5.1 to work out the matrix which represents a combined transformation.

Notes

Transformations will be limited to transforming the unit square.

Transformations will be restricted to those listed in 5.3.

A combined transformation may be referred to as a composite transformation.

Examples

1 The matrix $\mathbf{M} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Describe geometrically the transformation represented by

(a) **M**

(b) $\mathbf{M}^2$

2 Here are two transformations in the x - y plane:

A: Reflection in the x-axis

B: Rotation clockwise about the origin through 90º.

(a) Work out the single matrix which represents the composite transformation **A** followed by **B**.

(b) Does the composite transformation **B** followed by **A** give the same image of a shape as **A** followed by **B**?

Explain your answer.

3 The matrix **A** is such that $\mathbf{A}^2 = \mathbf{I}$ and $\mathbf{A} \neq \mathbf{I}$

By considering transformations in the x - y plane, work out a possible matrix for **A**.

4 Here are three transformations in the x - y plane:

A: Reflection in the x-axis

B: Reflection in the y-axis

C: Rotation about O through 180°.

Use matrix multiplication to prove that **C** is the same as **A** followed by **B**.

6 Geometry

6.1 Geometry

Assessment Guidance

Candidates should be able to

- understand perimeter
- recall and use the formula for area of a rectangle
- recall and use the formula $\frac{1}{2}$ × base × height for area of a triangle
- use the formula $\frac{1}{2}ab\sin C$ for area of a triangle
- recall and use formulae for circumference and area of a circle
- recall and use formulae for volume of a cube, a cuboid, prisms and pyramids
- use formulae for volume of a cone and of a sphere
- understand and use angle properties of parallel and intersecting lines
- understand and use angle properties of triangles and special types of quadrilaterals and polygons
- understand and use circle theorems.

Notes

Candidates will be expected to use exact values in terms of π in some questions.

Questions involving surface areas of 3D shapes may also be set.

Knowledge and understanding of the circle theorems in the Key Stage 4 Programme of Study will be expected.

Examples

1 The area of a circle of radius r is equal to the area of a semicircle of radius R.

Show that $R = r\sqrt{2}$

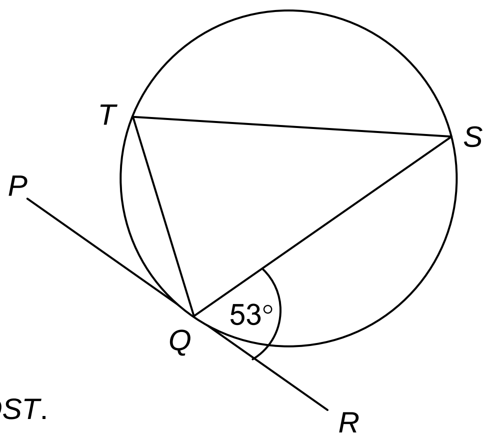
2 The diagram shows a circle.

PQR is a tangent.

Angle *SQR* = 53°

QT = 8 cm

TS = 10 cm



Not drawn accurately

Work out the area of the triangle *QST*.

6.2 Understand and construct geometrical proofs using formal arguments

Assessment Guidance

Candidates should be able to

- construct formal proofs using correct mathematical notation and vocabulary.

Notes

The use of angle facts and theorems listed in sections 3.7 and 6.1 are expected.

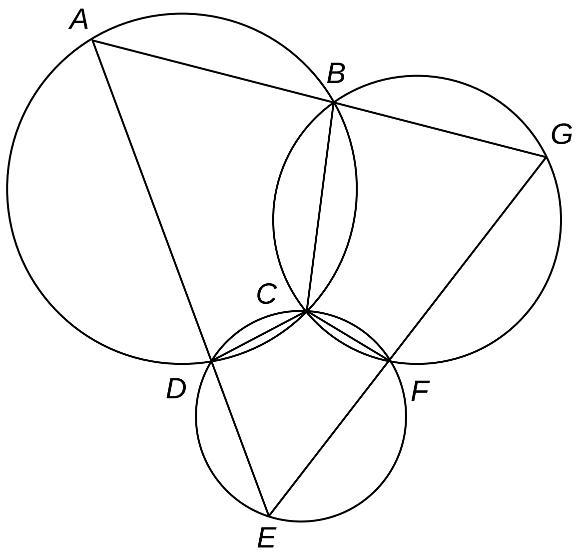
Angle facts used must always be accompanied by a reason.

Proofs of standard theorems will not be set.

Examples

1 *ABCD*, *CDEF* and *BCFG* are cyclic quadrilaterals.

ABG and *ADE* are straight lines.

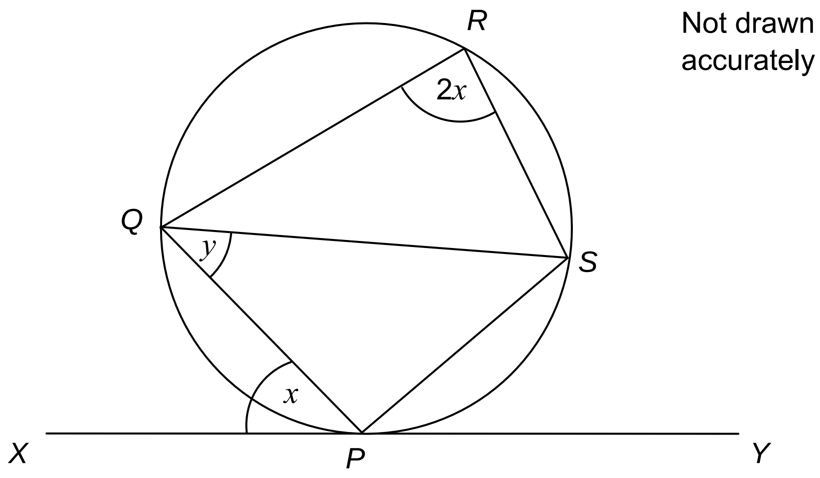


Not drawn accurately

Prove that *EFG* is a straight line.

2 In the diagram, PQRS is a cyclic quadrilateral.

XPY is a tangent to the circle.



Prove that $y = x$.

6.3 Sine and cosine rules in scalene triangles

Assessment Guidance

Candidates should be able to

- understand and use the formulae for sine rule and cosine rule.

Notes

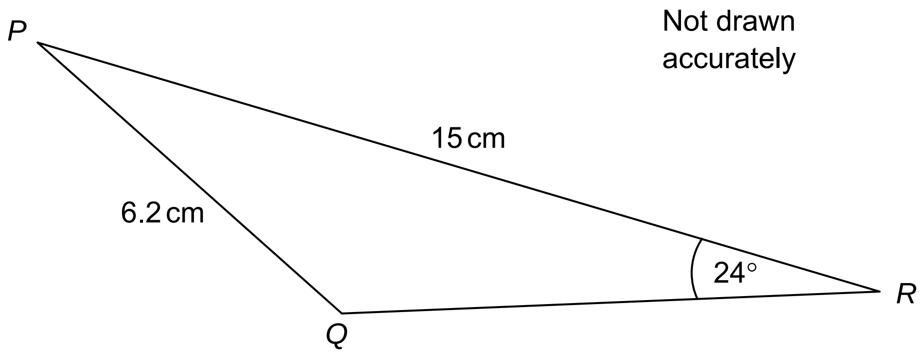
Knowledge and use of trigonometry in right-angled triangles is expected.

Trigonometry questions may be set on the non-calculator paper.

Angles will always be in degrees

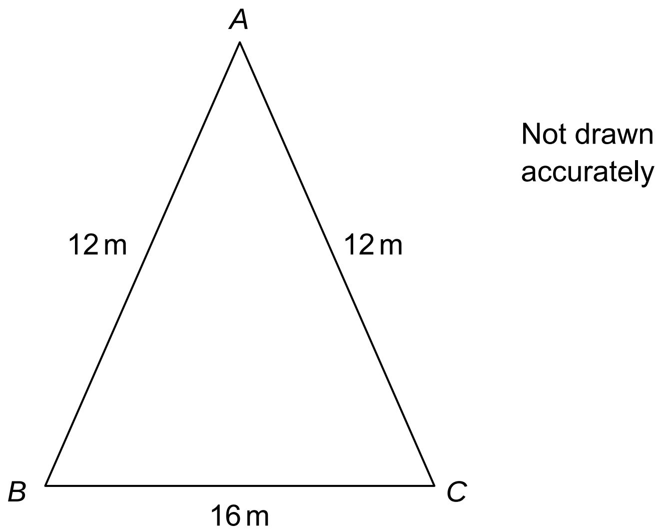
Examples

1 In this triangle angle *PQR* is obtuse.



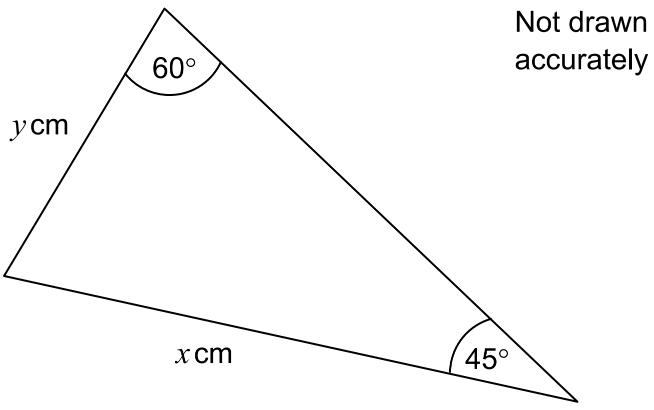
Work out angle *QPR*.

2 Triangle *ABC* is isosceles with *AB* = *AC*.



Work out angle *BAC*.

3 Here is a triangle.



Work out the exact value of $\frac{x}{y}$.

6.4 Use of Pythagoras' theorem in 2D and 3D

Assessment Guidance

Candidates should be able to

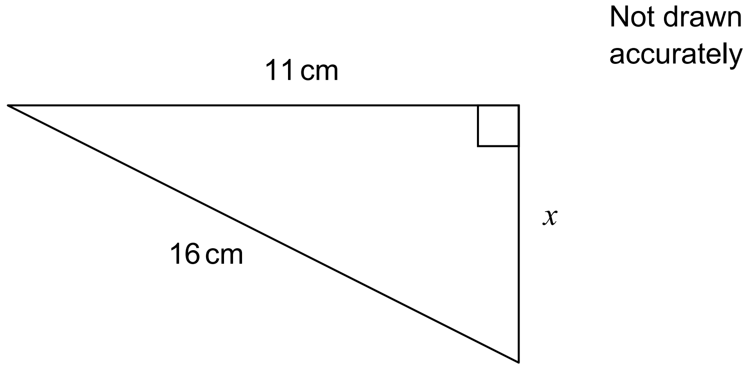
- work out any unknown side using two given sides
- identify appropriate right-angled triangles in 2 and 3 dimensional shapes and apply Pythagoras' theorem
- recognise and use Pythagorean triples.

Notes

Candidates will be expected to know the Pythagorean triples; 3, 4, 5; 5, 12, 13; 8, 15, 17; 7, 24, 25 and simple multiples of these.

Examples

1 Here is a triangle.



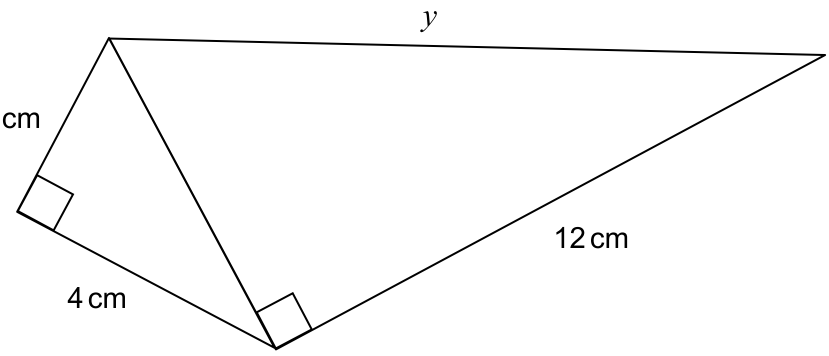
Work out the value of x.

2 A box is a cuboid measuring 30 cm by 10 cm by 4 cm.

Will a knitting needle of length 32 cm fit in the box?

You **must** show your working.

3



Work out the value of y.

6.5 Be able to apply trigonometry and Pythagoras' theorem to 2 and 3 dimensional problems

Assessment Guidance

Candidates should be able to

- identify appropriate right-angled triangles in 2 and 3 dimensional shapes and apply Pythagoras' theorem
- identify appropriate triangles in 2 and 3 dimensional shapes and apply trigonometry
- work out the angle between a line and a plane
- work out the angle between two planes
- understand and use bearings.

Notes

Candidates should understand angles of elevation and angles of depression.

Angles will always be in degrees.

Questions on ratio using trig methods will be set, but candidates may use similar triangles where appropriate.

Examples

1 A ship travels 15 km due North from port *C*.

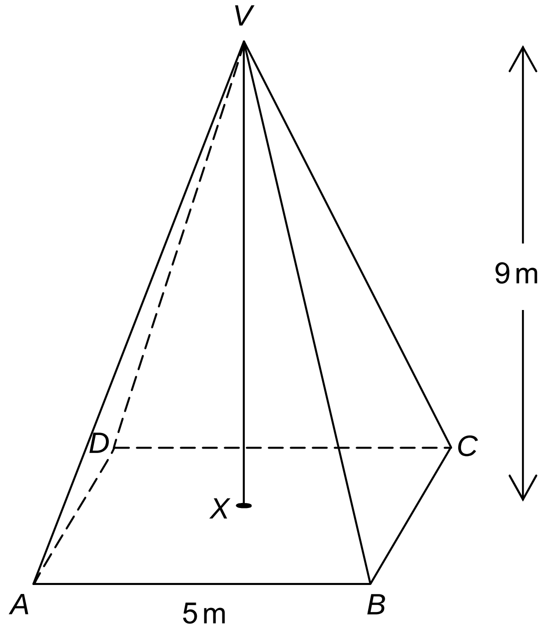
It then travels 18 km due East to port *D*.

Work out the bearing of *D* from *C*.

2 A pyramid has a square base *ABCD* of side 5 metres.

The vertex *V* is directly above the centre of the base *X*.

The height of the pyramid is 9 metres.



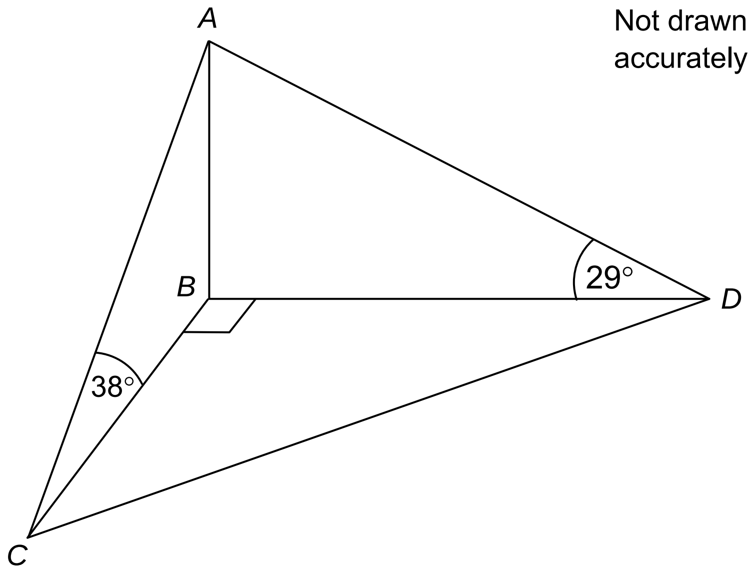
Work out the angle between the planes *ABCD* and *VAB*.

3 The diagram shows a tower *AB.*

The angle of elevation of the top of the tower from point *C,* due South of the tower, is 38°

The angle of elevation of the top of the tower from point *D,* due East of the tower, is 29°

The distance from *C* to *D* is 50 metres.



Work out the height of the tower, *AB*.

6.6 Sketch and use graphs of: $y = \sin x$, $y = \cos x$ and $y = \tan x$ for $0° \leq x \leq 360°$

Assessment Guidance

Candidates should be able to

- understand and use the properties of the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ for $0° \leq x \leq 360°$
- sketch and use the graphs to solve problems.

Notes

Candidates should be familiar with the asymptotes on the graph of $y = \tan x$ but the word asymptote will not be used in a question.

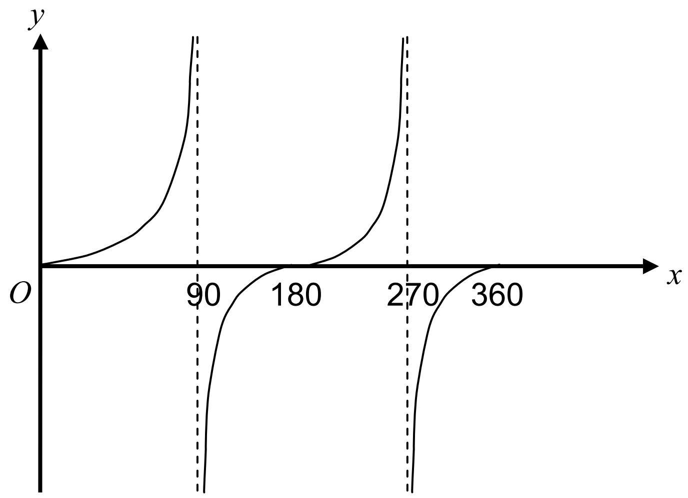
Examples

1 Sketch the graph of $y = \cos x$ for $0° \leq x \leq 360°$

(a) State the coordinates of the points of intersection with the axes.

(b) Write down the minimum value of y.

2 A sketch of $y = \tan x$ for $0° \leq x \leq 360°$ is shown.



(a) How many solutions to $\tan x = 2$ are between 0° and 180° ?

(b) How many solutions to $\tan x = –0.4$ are between 0° and 270° ?

(c) Write down the solutions of $\tan x = 0$ for $0° \leq x \leq 360°$

6.7 Be able to use the definitions of sin θ, cos θ and tan θ for any positive angle up to 360°

Assessment Guidance

Candidates should be able to

- understand and use the properties of the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ for $0° \le x \le 360°$
- sketch and use the graphs to solve problems.

Notes

Angles measured anticlockwise will be taken as positive.

Questions may be set on the non-calculator paper.

Examples (both non-calculator)

1 Given that $\cos 30° = \frac{\sqrt{3}}{2}$, work out a value of x between 90° and 360° for which $\cos x = \frac{\sqrt{3}}{2}$

2 Given that $\sin 210° = -\frac{1}{2}$, work out the value of sin 150°.

6.8 Knowledge and use of 30°, 60°, 90° triangles and 45°, 45°, 90° triangles

Assessment Guidance

Candidates should be able to

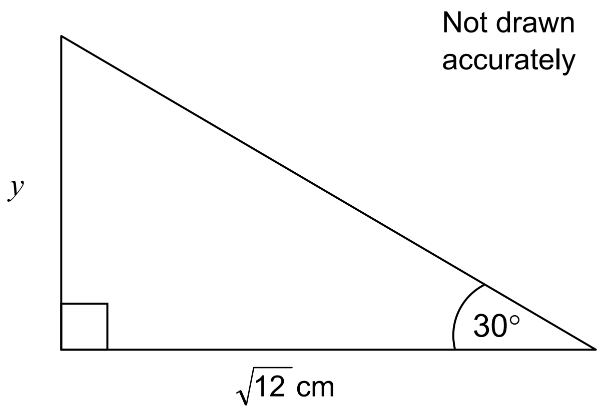
- recall or work out the exact values of the trigonometric ratios for angles 30°, 45° and 60°.

Notes

The exact values may be quoted unless the question specifically asks for a derivation.

Examples

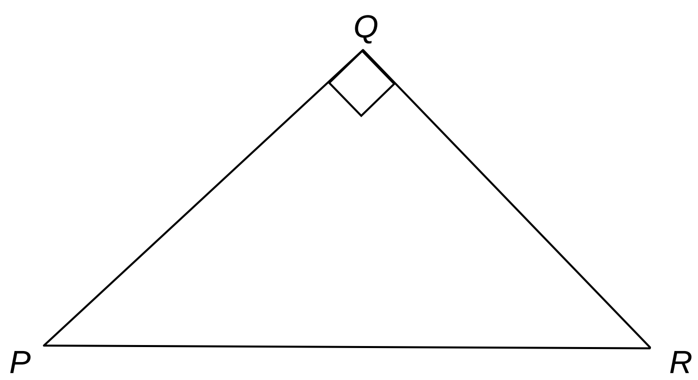
1 Here is a triangle.



Show that y is an integer.

2 Here is an isosceles triangle.

$PQ = QR$



Explain why tan 45° = 1.

6.9 Use of $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta \equiv 1$

Assessment Guidance

Candidates should be able to

- use the identities to simplify expressions
- use the identities to prove other identities
- use the identities in solution of equations.

Notes

The identity symbol should be known and understood.

Examples (both non-calculator)

1 Given that $\sin\theta = \dfrac{1}{3}$ and θ is obtuse, work out the exact value of $\cos\theta$.

2 Solve $2\sin x = \cos x$ for $0° \le x \le 90°$

3 Prove $\dfrac{1}{\cos^2 x} - \tan^2 x \equiv 1$

6.10 Solution of simple trigonometric equations in given intervals

Assessment Guidance

Candidates should be able to

- work out all solutions in a given interval
- rearrange equations including the use of the identities from section 6.9
- use factorisation.

Notes

Questions will always use single angles.

Solutions less than 0° or greater than 360° will not be required, but as calculators may give negative solutions, candidates should use the cyclic properties of the trigonometric graphs in 6.6 to give solutions in the range $0° \le \theta \le 360°$.

Angles will always be in degrees.

Questions may be set on the non-calculator paper but solutions will involve angles from section 6.8.

Examples

1 Solve $\sin x = 0.5$ for $0° \le x \le 360°$

2 Solve $\tan^2 \theta - 2\tan\theta = 0$ for $0° \le \theta \le 180°$

3 Solve $4\cos x = -3$ for $0° \le x \le 360°$

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