

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

GCSE MATHEMATICS

Example-Problem Past Paper

H

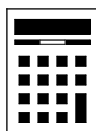
Higher Tier Paper 2 Calculator

June 2023

Materials

For this paper you must have:

- a calculator
- mathematical instruments.



Instructions

- Engage with the fully-worked solutions in full before attempting the shadow questions.
- Explain the fully-worked solutions to yourself, anticipating the next steps in the worked solutions, making links between the problems and the mathematics used to solve them.
- Apply the methods learnt from the fully-worked solutions to the shadow questions, writing down all workings in the spaces provided. Your thought process is important.
- Do all rough work in this book.

Information

- The marks for questions are shown in brackets.
- You may ask for more answer paper, graph paper and tracing paper.
- You should ask your teacher for help on a question if you do not understand a part of the fully-worked solution. Remember to be specific, understanding why the step was completed, rather than simply getting the correct answer.

Advice

In all calculations, show clearly how you work out your answer.

Where a calculator has been used, show clearly what you entered into the calculator.

Answer **all** questions in the spaces provided.

- 1 Write $30 : 12$ in the form $n : 1$

[1 mark]

$$\begin{array}{c} \div 12 \quad \downarrow \quad 30 : 12 \quad \downarrow \quad \div 12 \\ \quad \quad 2.5 : 1 \quad \quad \end{array}$$

Answer 2.5 : 1

- 2 Four consecutive triangular numbers are

$$\begin{array}{ccccccc} & & +4 & & +6 & & \\ & \nearrow & & \nearrow & & \nearrow & \\ 6 & & 10 & & 15 & & 21 \\ & \nwarrow & & \nwarrow & & \nwarrow & \\ & & +5 & & +7 & & \end{array}$$

Write down the next triangular number.

[1 mark]

$$21 + 7 = 28$$

Answer 28

Answer **all** questions in the spaces provided.

Do not write
outside the
box

- 1** Write $28 : 8$ in the form $n : 1$

[1 mark]

Answer _____ : 1

- 2** Four consecutive terms from the Fibonacci sequence are 3 5 8 13
Write down the next term.

[1 mark]

Answer _____

Turn over ►

- 3 Write down the reciprocal of $\frac{4}{7}$ [1 mark]

Answer $\frac{7}{4}$

- 4 The price of a toy increases by 12.5% to £19.53
Work out the **original** price of the toy. [2 marks]

$$\begin{array}{r|l} 112.5\% & \text{£}19.53 \\ \hline 100\% & \text{£}17.36 \end{array} \quad \begin{array}{l} \nearrow \times 1.125 \\ \searrow \div 1.125 \end{array}$$

Answer £ 17.36

- 3 Write down the reciprocal of $\frac{5}{8}$ [1 mark]

Answer _____

- 4 The price of a necklace increases by 37.5% to £38.17
Work out the **original** price of the necklace. [2 marks]

Answer £ _____

5

Jess saves 2p, 5p and 10p coins.

She has

- 45 10p coins
- 8 times as many 2p coins as **10p coins**
- £17.70 in total.

Work out total **value** of 2p coins : total **value** of 5p coins

Give your answer in its simplest form.

[4 marks]

$$45 \times 10p = 450p = £4.50$$

$$8 \times 45 = 360 \quad 360 \times 2p = 720p = £7.20$$

$$4.50 + 7.20 = 11.70$$

$$17.70 - 11.70 = £6.00 \text{ in } 5p \text{ coins}$$

$$£7.20 : £6.00$$

$$\div 10 \left(\begin{array}{l} 720 : 600 \end{array} \right) \div 10$$

$$\div 12 \left(\begin{array}{l} 72 : 60 \\ 6 : 5 \end{array} \right) \div 12$$

Answer 6 : 5

She has

- Work out total **value** of 2p coins : total **value** of 5p coins

[4 marks]

Answer _____ :

- 6 (a) Part of a regular polygon is shown.



Not drawn
accurately

Assume that the polygon is an octagon.

Work out the size of an **exterior** angle.

[2 marks]

Exterior angles sum to 360°

$$\frac{360^\circ}{8} = 45^\circ$$

Answer 45 $^\circ$

- 6 (b) In fact, the polygon has **more** sides than an octagon.

What does this mean about the size of an exterior angle?

Tick **one** box.

$$\frac{360}{9} = 40^\circ$$

$$\frac{360}{10} = 36^\circ$$

[1 mark]

☐

It is more than the answer to part (a)

☐

It is the same as the answer to part (a)

☒

It is less than the answer to part (a)

☐

It could be any of the above

- 6 (a) Part of a regular polygon is shown.



Not drawn
accurately

Assume that the polygon is a hexagon.

Work out the size of an **exterior** angle.

[2 marks]

Answer _____ °

- 6 (b) In fact, the polygon has **more** sides than a hexagon.

What does this mean about the size of an exterior angle?

Tick **one** box.

[1 mark]

☐

It is more than the answer to part (a)

☐

It is the same as the answer to part (a)

☐

It is less than the answer to part (a)

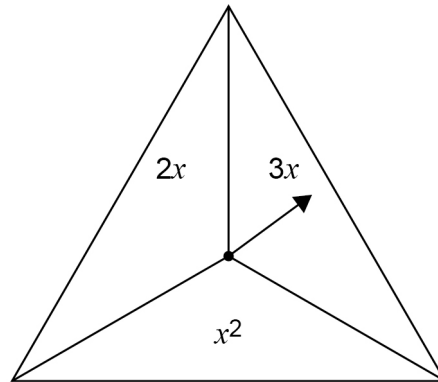
☐

It could be any of the above

7

In a game,

- an ordinary fair six-sided dice is rolled
- the fair spinner shown is spun.



The score is the dice number **substituted** into the spinner expression.

7 (a) Complete the table to show all of the possible scores.

[2 marks]

	1	2	3	4	5	6
$2x$	2	4	6	8	10 ✓	12 ✓
$3x$	3	6	9	12 ✓	15 ✓	18 ✓
x^2	1	4	9	16 ✓	25 ✓	36 ✓

- 7 (b) A player wins the game if their score is 10 or more.

Work out the probability that they win the game.

[1 mark]

8 scores win

Answer $\frac{8}{18} = \frac{4}{9}$

- 7 (c) The game is played 711 times.

Estimate the number of games that are won.

[2 marks]

$\frac{4}{9} \times 711 = 316$

Answer 316

8 $(a - 3)x^2 + 2b \equiv 5x^2 + 12$

Work out the values of a and b .

[2 marks]

$a - 3 = 5$
 $+3 \quad +3$
 $a = 8$

$2b = 12$
 $\div 2 \quad \div 2$
 $b = 6$

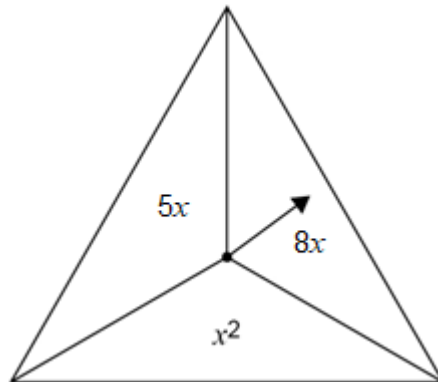
$a = 8 \quad b = 6$

Turn over ►

7

In a game,

- an ordinary fair six-sided dice is rolled
- the fair spinner shown is spun.



The score is the number on the dice **substituted** into the spinner expression.

7 (a) Complete the table to show all of the possible scores.

[2 marks]

	1	2	3	4	5	6
$5x$						30
$8x$		16				
x^2				16		

- 7 (b)** A player wins the game if their score is 30 or more.

Work out the probability that they win the game.

[1 mark]

Answer _____

- 7 (c)** The game is played 756 times.

Estimate the number of games that are won.

[2 marks]

Answer _____

8 $(a - 5)x^2 + 4b \equiv 3x^2 + 20$

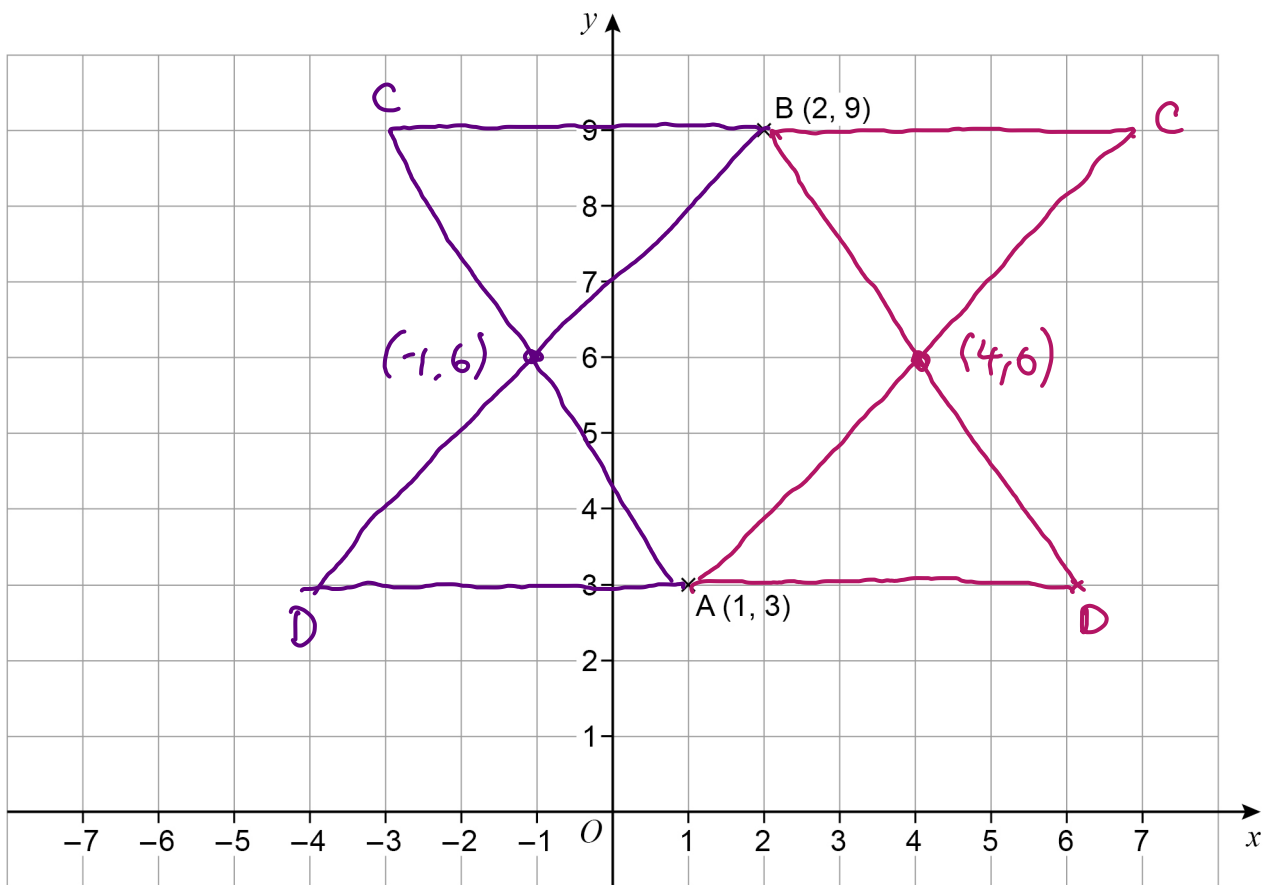
Work out the values of a and b .

[2 marks]

$a =$ _____ $b =$ _____

9

A (1, 3) and B (2, 9) are points on a centimetre grid.



ABCD is a parallelogram.

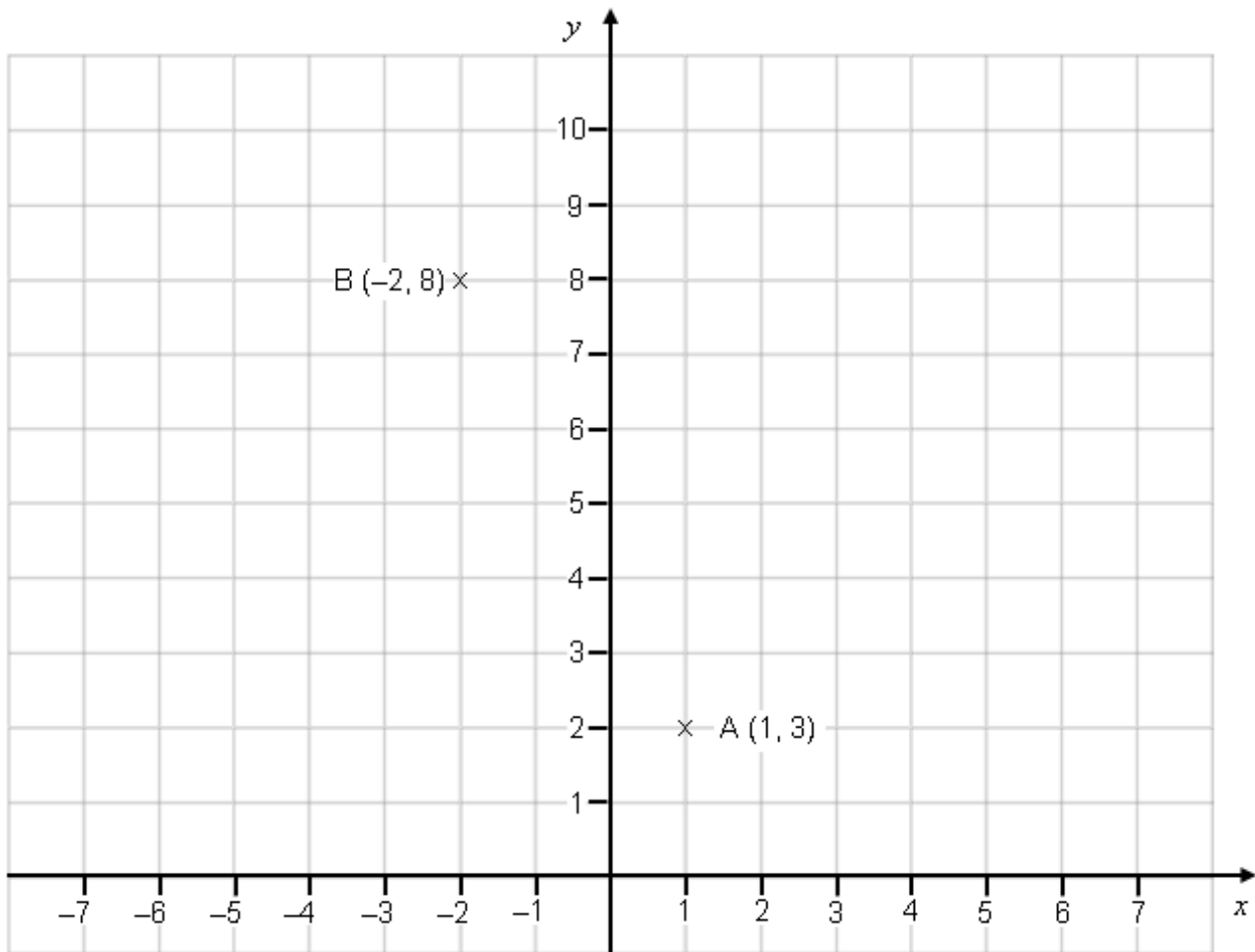
AD and BC are **horizontal** and each has length 5 cm

The diagonals of ABCD cross at E.

Work out the **two** possible pairs of coordinates of E.**[4 marks]**

Answer (4 , 6) and (-1 , 6)

- 9 A (1, 2) and B (−2, 8) are points on a centimetre grid.



ABCD is a parallelogram.

AD and BC are **horizontal** and each has length 5 cm

The diagonals of ABCD cross at E.

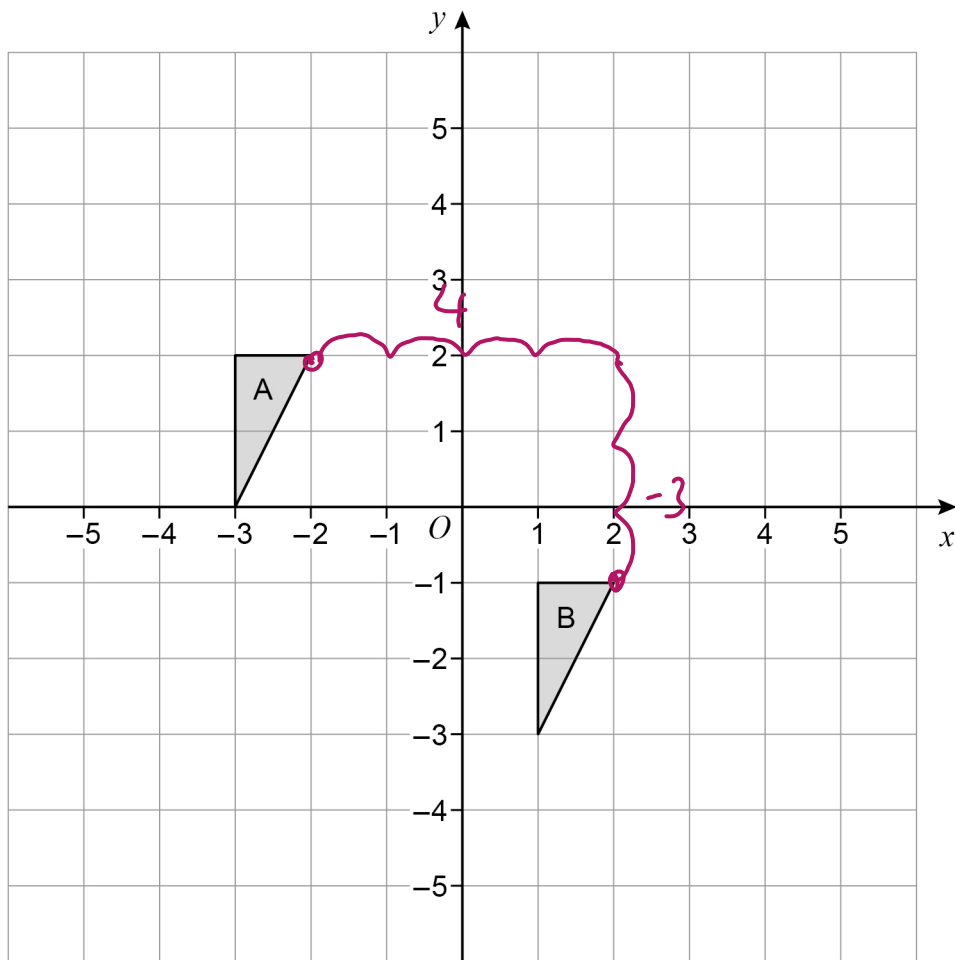
Work out the **two** possible pairs of coordinates of E.

[4 marks]

Answer (_____ , _____) and (_____ , _____)

10

Write down the translation vector that maps shape A onto shape B.

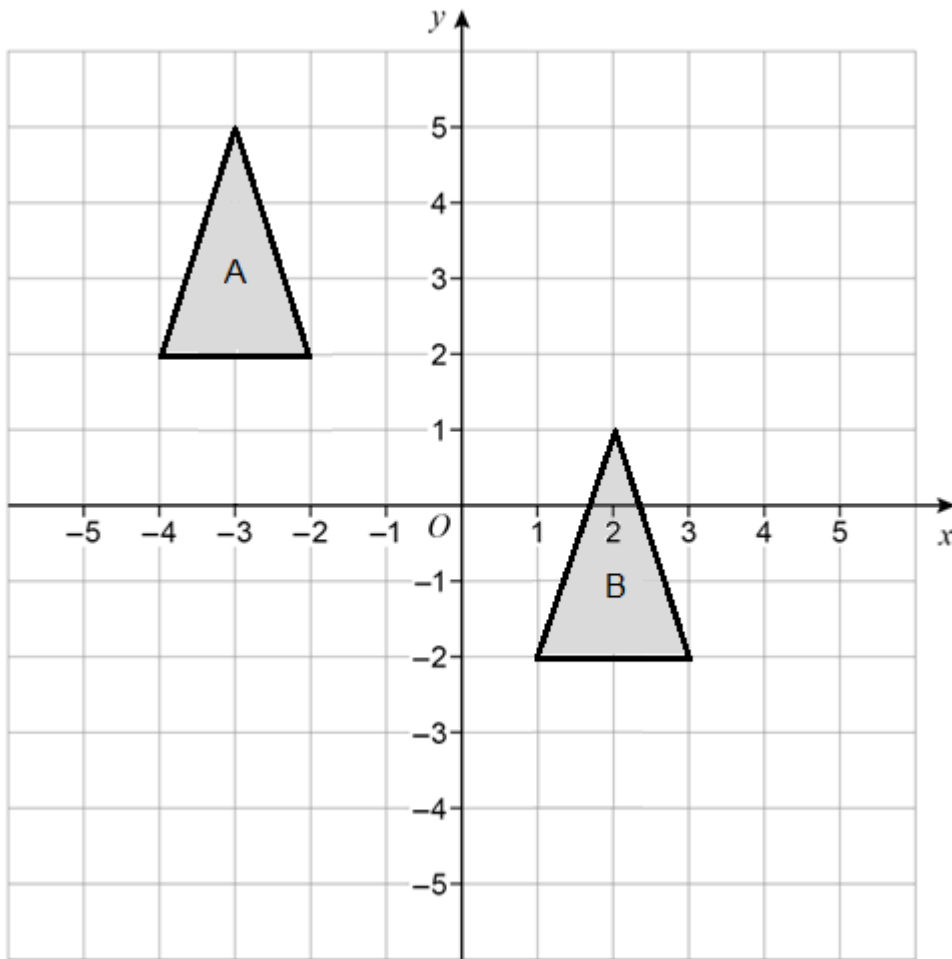
[2 marks]

Answer _____

 $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

10

Write down the translation vector that maps shape A onto shape B.

[2 marks]

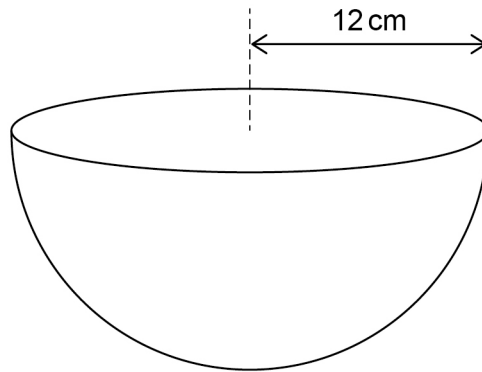
Answer _____

Turn over ►

11

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

A bowl is a hemisphere with radius 12 cm



Water is poured into the bowl
at a rate of 325 cm^3 per second
for 8 seconds.

Does the water fill **more than** 70% of the bowl?

You **must** show your working.

[4 marks]

$$\text{Water} : 325 \times 8 = 2600 \text{ cm}^3$$

$$\text{Volume of hemisphere} : \frac{1}{2} \text{ of } \frac{4}{3} \pi r^3 = \frac{2}{3} \pi r^3$$

$$\frac{2}{3} \times \pi \times 12^3 = 3619.114737$$

$$70\% \text{ of volume} : 0.7 \times 3619.114737 = 2533.380316$$

Yes.

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

A diagram of a hemispherical bowl. A vertical dashed line extends from the center of the circular rim to the bottom of the bowl. A horizontal double-headed arrow indicates the radius from this dashed line to the rim, labeled "9 cm".

Does the water fill **more than** 80% of the bowl?
You **must** show your working.

[4 marks]

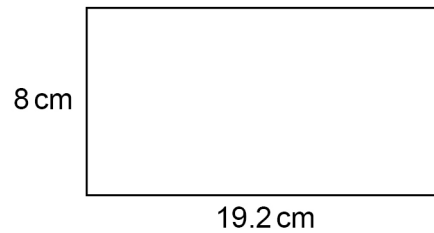
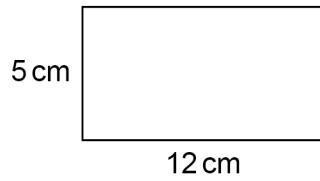
4

Turn over ►

- 12** Show that these two rectangles are similar.

[2 marks]

Not drawn
accurately



$$8 \div 5 = 1.6$$

$$19.2 \div 12 = 1.6$$

- 13** A factory packs x boxes of teabags per hour.
Each box contains 80 teabags.

Show that the factory packs $\frac{4x}{3}$ teabags per minute.

[2 marks]

$$80 \times x = 80x \text{ teabags per hour}$$

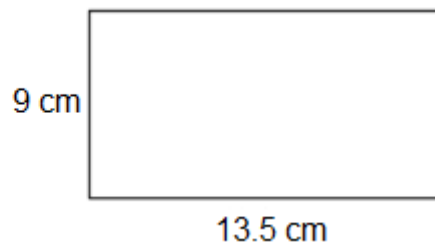
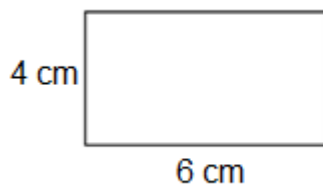
$$\downarrow \div 60$$

$$\frac{80x}{60} \text{ teabags per minute} = \frac{4x}{3}$$

- 12** Show that these two rectangles are similar.

[2 marks]

Not drawn
accurately



- 13** A factory packs x boxes of plasters per hour.
Each box contains 100 plasters.

Show that the factory packs $\frac{5x}{3}$ plasters per minute.

[2 marks]

Turn over ►

14

A company has 123 employees.

Information about their hourly rates of pay is shown in the table.

Hourly rate, £ p	Number of employees
$10 \leq p < 14$	66
$14 \leq p < 20$	32 $(\frac{1}{2})A$
$20 \leq p < 40$	15 A
$40 \leq p < 100$	10 A
Total = 123	

The owner of the company uses the data to make two statements.

Statement A

“Over 30% of employees have an hourly rate that is more than £17”

Statement B

“The average hourly rate of pay is more than £20”

14 (a) Show working that supports **Statement A**.

[3 marks]

$$10 + 15 + 16 = 41$$

$$\frac{41}{123} \times 100 = 33.3\%$$

14 (b) Why might **Statement A** not be true?

[1 mark]

The 32 employees in the $14 \leq p < 20$ interval might not be evenly distributed. If more earn £14, this would make statement A false.

14 (c) Work out an estimate of the mean to support **Statement B**.

[3 marks]

mp f

$$10 \times 66 = 660$$

$$17 \times 32 = 544$$

$$30 \times 15 = 450$$

$$70 \times 10 = 700$$

$$\underline{2354}$$

$$\frac{2354}{123} = 19.13821138$$

$$£19.14 > £18.$$

14 (d) Why is the mean **not** the best average to represent the data?

[1 mark]

Extreme values (hourly pay of £100) dramatically increase the mean

Turn over ►

14

A company has 113 employees.

Information about their hourly rates of pay is shown in the table.

Hourly rate, £ p	Number of employees
$8 \leq p < 12$	56
$12 \leq p < 20$	28
$20 \leq p < 40$	17
$40 \leq p < 60$	12
	Total = 113

The owner of the company uses the data to make two statements.

Statement A

“Over 35% of employees have an hourly rate that is more than £16”

Statement B

“The average hourly rate of pay is more than £18”

14 (a) Show working that supports **Statement A**.

[3 marks]

14 (b) Why might **Statement A** not be true?

[1 mark]

14 (c) Work out an estimate of the mean to support **Statement B**.

[3 marks]

14 (d) Why is the mean **not** the best average to represent the data?

[1 mark]

15

Expand $(x^2 - 9xy)(2x + 5y)$

[2 marks]

$$\begin{array}{r|l}
 x^2 - 9xy & \\
 \hline
 2x & 2x^3 - 18x^2y \\
 +5y & 5x^2y - 45xy^2 \\
 \hline
 & 2x^3 - 13x^2y - 45xy^2
 \end{array}$$

Answer $2x^3 - 13x^2y - 45xy^2$

16

Line A

has equation $y = ax - 1$

passes through the point (7, 13)

Line B has equation $5y - 3x = 4$

Show that line A has a greater gradient than line B.

[3 marks]

$$y = ax - 1 \quad (7, 13) \rightarrow 13 = 7a - 1$$

$$+1 \quad +1$$

$$14 = 7a$$

$$\div 7 \quad \div 7$$

$$2 = a$$

$$y = 2x - 1$$

$$m_A = 2$$

$$5y - 3x = 4$$

$$+3x \quad +3x$$

$$5y = 3x + 4$$

$$\div 5 \quad \div 5$$

$$y = \frac{3}{5}x + \frac{4}{5}$$

$$m_B = \frac{3}{5}$$

$$2 > \frac{3}{5}$$

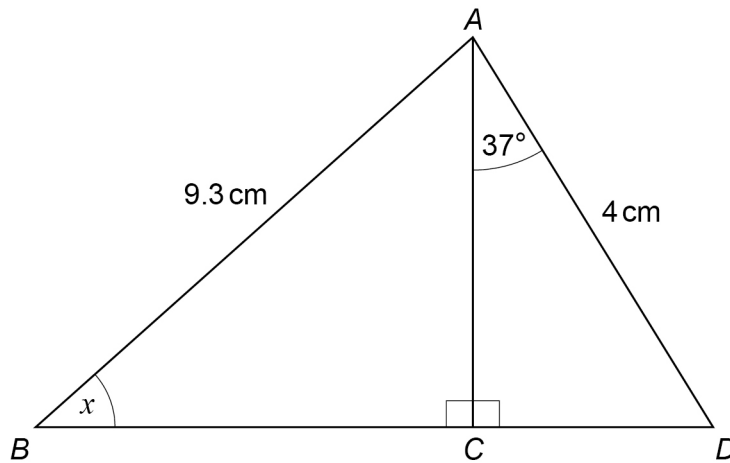
[2 marks]

16 Line A
has equation $y = ax - 5$
passes through the point (9, 22)

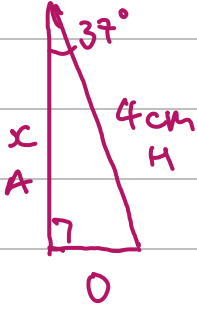
Show that line A has a greater gradient than line B.

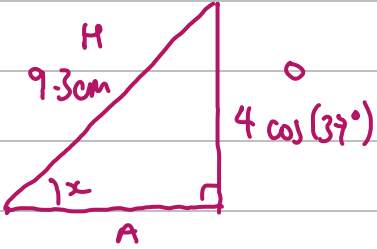
[3 marks]

17

Not drawn
accuratelyWork out the size of angle x .

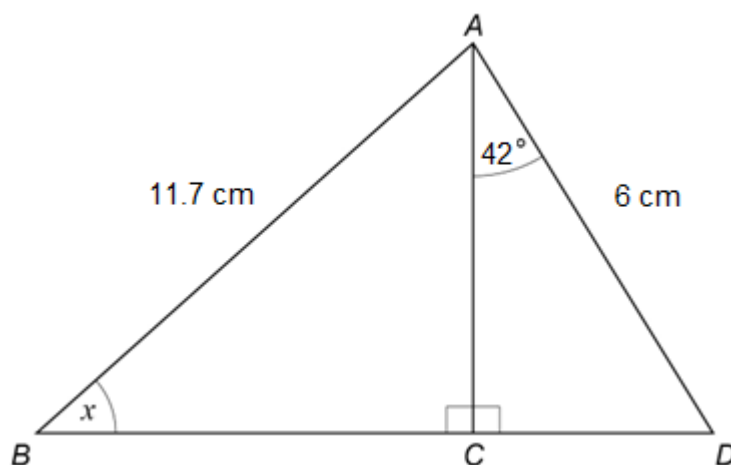
[4 marks]


 $\cos \theta = \frac{A}{H}$
 $\cos 37^\circ = \frac{x}{4}$
 $(\cos 37^\circ) \times 4 = x = 4 \cos(37)^\circ$


 $\sin \theta = \frac{O}{H}$
 $\sin x^\circ = \frac{4 \cos(37)}{9.3}$
 $x^\circ = \sin^{-1}\left(\frac{4 \cos(37)}{9.3}\right) = 20.09020484$
 ≈ 20.1

$$x = 20.1^\circ$$

Not drawn
accurately



[4 marks]

$x =$ _____

4

Turn over ►

18

Rearrange $y = \frac{x+8}{x}$ to make x the subject.

[3 marks]

$$y = \frac{x+8}{x}$$

$$xy = x+8$$

$$xy - x = 8$$

$$x(y-1) = 8$$

$$\div (y-1) \quad \div (y-1)$$

$$x = \frac{8}{y-1}$$

Answer $x = \frac{8}{y-1}$

18 Rearrange $z = \frac{xy + 4}{x}$ to make x the subject.

[3 marks]

Answer _____

19

Here are the first four terms of a quadratic sequence.

3 20 47 84

Work out an expression for the n th term of the sequence.**[4 marks]**

$$\begin{array}{ccccccc}
 3 & & 20 & & 47 & & 84 \\
 & \nearrow +17 & & \nearrow +27 & & \nearrow +37 & \\
 & & & \nearrow +10 & & \nearrow +10 & \\
 & & & & & & 10 \div 2 = 5
 \end{array}$$

$$\begin{array}{ccccccc}
 (5n^2) & 5 & 20 & 45 & 80 & & \\
 \frac{-2}{3} & \frac{0}{20} & \frac{+2}{47} & \frac{+4}{84} & & (2n-4) & \\
 & & & & & &
 \end{array}$$

$$(5n^2) + (2n - 4)$$

Answer $5n^2 + 2n - 4$

Here are the first four terms of a quadratic sequence.

Work out an expression for the n th term of the sequence.

[4 marks]

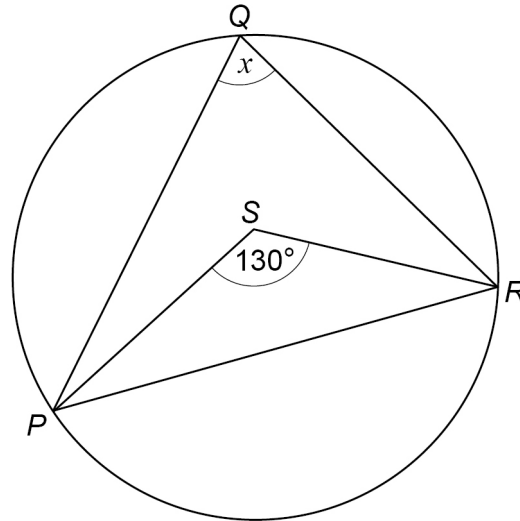
Answer

4

Turn over ►

- 20 (a)** P , Q and R are points on a circle.
 S is a point inside triangle PQR .

Not drawn
accurately



Assume that S is the centre of the circle.

Work out the size of angle x .

[1 mark]

$$130 \div 2 = 65$$

$$x = 65^\circ$$

- 20 (b)** In fact, the centre of the circle is on PS but **not** at S .

What does this mean about the size of angle x ?

Tick **one** box.

[1 mark]

☐

It is the same as the answer to part (a)

☒

It is greater than the answer to part (a)

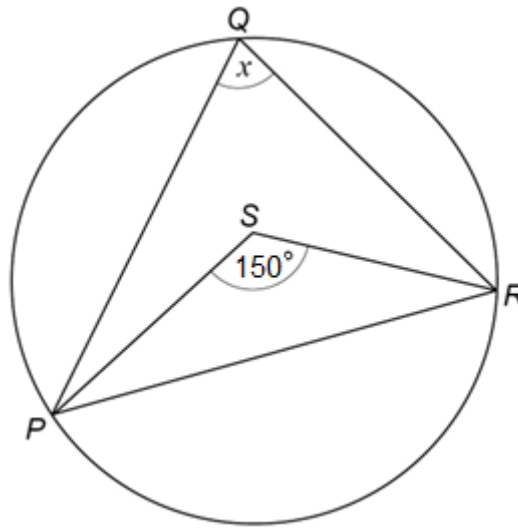
☐

It is smaller than the answer to part (a)

☐

It is impossible to tell

- 20 (a)** P , Q and R are points on a circle.
 S is a point inside triangle PQR .



Not drawn
accurately

Assume that S is the centre of the circle.

Work out the size of angle x .

[1 mark]

$$x = \underline{\hspace{2cm}}^{\circ}$$

- 20 (b)** In fact, S is not the centre of the circle.

What does this mean about the size of angle x ?

Tick **one** box.

[1 mark]

☐

It is the same as the answer to part (a)

☐

It is greater than the answer to part (a)

☐

It is smaller than the answer to part (a)

☐

It could be bigger or smaller than the answer to part (a)

Turn over ►

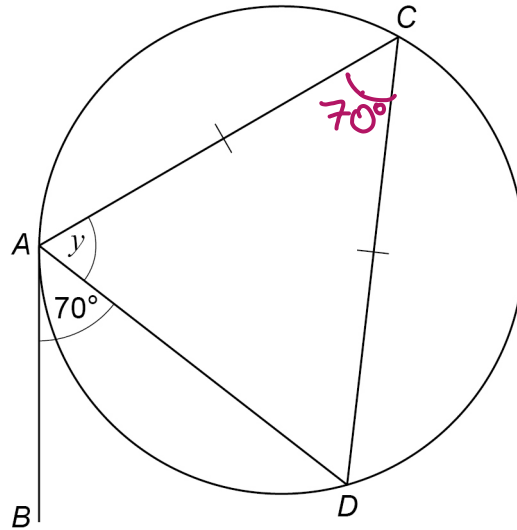
20 (c) For a different circle,

AB is a tangent at A

C and D are on the circumference of the circle

$AC = CD$

Not drawn
accurately



Here is Simon's method to work out the size of angle y .

Angle $ADC = 70^\circ$ (alternate segment theorem)
Therefore $y = 70^\circ$ (angles in an isosceles triangle)

Is he correct?

Give a reason for your answer.

[1 mark]

No, Angle $ADC \neq 70^\circ$, angle $ACD = 70^\circ$

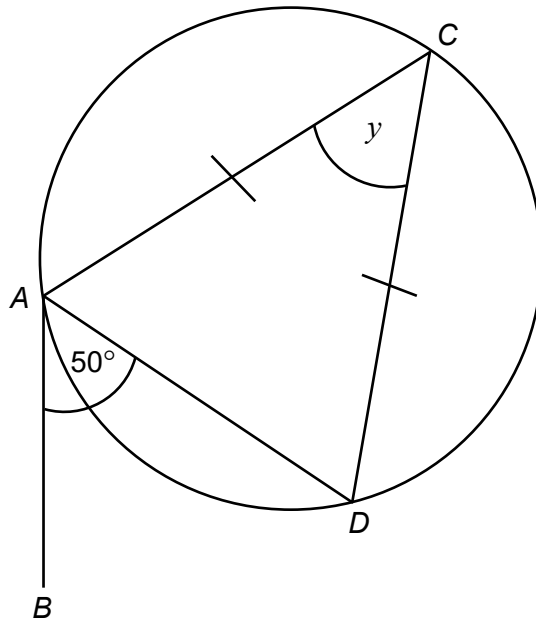
20 (c) For a different circle,

AB is a tangent at A

C and D are on the circumference of the circle

$AC = CD$

Not drawn
accurately



Here is Ollie's method to work out the size of angle y .

Angle $ADC = 50^\circ$ (alternate angles are equal)

Angle $CAD = 50^\circ$ (angles in an isosceles triangle)

Therefore $y = 80^\circ$ (angles in a triangle)

Is he correct?

Give a reason for your answer.

[1 mark]

21

Magana decides to put £500 into an account that pays compound interest.
She wants to have **at least** £560 in the account after 3 years.

Work out to 1 decimal place the **minimum** annual interest rate she needs.

[3 marks]

$$500 \times x^3 > 560$$

$$x^3 > \frac{560}{500}$$

$$x^3 > 1.12$$

$$x > \sqrt[3]{1.12} = 1.03849882 = 103.849882\%$$

Needs 103.9%

Answer 3.9 %

Asmae decides to put £2500 into an account that pays compound interest. She wants to have **at least** £3200 in the account after 5 years.

[3 marks]

Answer %

3

Shadow paper based on June 2023 question paper

- 22** An approximate value of a root of an equation, x , can be found using the iterative formula

$$x_{n+1} = \sqrt[3]{5(x_n)^2 - 2x_n - 3}$$

The starting value is $x_1 = 4$

- 22 (a)** Work out the values of x_2 and x_3

[2 marks]

$$x_2 = \sqrt[3]{5(x_1)^2 - 2(x_1) - 3} = \sqrt[3]{5(4)^2 - 2(4) - 3}$$

$$x_3 = \sqrt[3]{5(x_2)^2 - 2(x_2) - 3} = \sqrt[3]{5(\text{Ans})^2 - 2(\text{Ans}) - 3}$$

$$x_2 = 4.10156593$$

$$x_3 = 4.177641648$$

- 22 (b)** By continuing the iteration, show that the value of x is more than 4.25

[1 mark]

$$x_4 = 4.234116777$$

$$x_5 = 4.275769447$$

$$x_6 = 4.306345111$$

- 22** An approximate value of a root of an equation, x , can be found using the iterative formula

$$x_{n+1} = \sqrt[3]{7(x_n)^2 - 4x_n - 5}$$

The starting value is $x_1 = 5$

- 22 (a)** Work out the values of x_2 and x_3

[2 marks]

$$x_2 = \underline{\hspace{10cm}}$$

$$x_3 = \underline{\hspace{10cm}}$$

- 22 (b)** By continuing the iteration, show that the value of x is more than 5.85

[1 mark]

23

Here are three sets of cards.

Set A 1 1 3 5 5 5 6 8

Set B 1 2 4 6 8 8 9

Set C 3 4 5 6

In a game, a player has two options.

Option 1

Pick two cards from Set A

Option 2

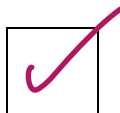
Pick one card from Set B
and
pick one card from Set C

The cards are picked at random.

The player wins if the total of their two cards is exactly 10

Which option gives a better chance of winning?

Option 1



Option 2



Show working to support your answer.

[4 marks]

Option 1 - 2 '5's $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

Option 2 - 4 and 6 $\frac{1}{7} \times \frac{1}{4} = \frac{1}{28}$

or

6 and 4 $\frac{1}{7} \times \frac{1}{4} = \frac{1}{28}$

$= \frac{2}{28} = \frac{1}{14}$

23

Here are three sets of cards.

Set A

1	2	3	3	6	6	6	8	8	8
---	---	---	---	---	---	---	---	---	---

Set B

1	1	2	4	7	7	8	8	10	10
---	---	---	---	---	---	---	---	----	----

Set C

3	3	3	6	6	7	8	8	9
---	---	---	---	---	---	---	---	---

In a game, a player has two options.

Option 1

Pick two cards from Set A

Option 2

Pick one card from Set B
and
pick one card from Set C

The cards are picked at random.

The player wins if the total of their two cards is exactly 12

Which option gives a better chance of winning?

Option 1

Option 2

Show working to support your answer.

[4 marks]

4

Turn over ►

24

 $a = 45$ to the nearest integer $b = 70$ to 1 significant figureWork out the **upper bound** for $6a^2 - b^2$ You **must** show your working.**[3 marks]**

$$44.5 \leq a < 45.5$$

$$65 \leq b < 75$$

$$\text{upper bound} = 6 \times a_{\max}^2 - b_{\min}^2$$

$$= 6 \times 45.5^2 - 65^2$$

$$= 8196.5$$

Answer 8196.5

24

 $a = 65$ to the nearest integer $b = 30$ to 1 significant figureWork out the **upper bound** for $2a^2 - b^2$ You **must** show your working.**[3 marks]**

Answer _____

25

Show that $\frac{x-7}{x-4} + \frac{x+7}{x+4}$

simplifies to $\frac{ax^2-b}{x^2-16}$ where a and b are integers.

[3 marks]

$$\frac{x-7}{x-4} + \frac{x+7}{x+4}$$

$$= \frac{(x-7)(x+4)}{(x-4)(x+4)} + \frac{(x+7)(x-4)}{(x+4)(x-4)}$$

$$= \frac{x^2-3x-28}{x^2-16} + \frac{x^2+3x-28}{x^2-16}$$

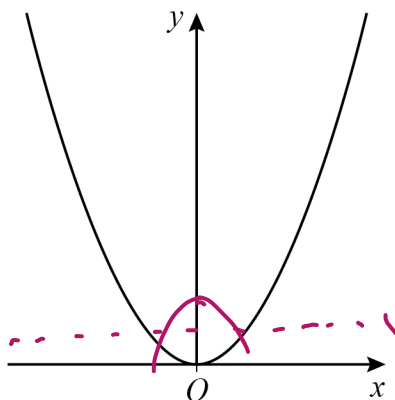
$$= \frac{2x^2-56}{x^2-16} \quad a=2 \quad b=56$$

Show that $\frac{x-5}{x-2} + \frac{x+5}{x+2}$

simplifies to $\frac{ax^2-b}{x^2-4}$ where a and b are integers.

Turn over ►

26 Here is a sketch of $y = x^2$



26 (a) The minimum point of $y = x^2$ is at $(0, 0)$

Write down the coordinates of the minimum point of $y = x^2 - 3$

[1 mark]

Answer (0 , -3)

26 (b) The graph $y = x^2$ is reflected in the line $y = 1$

Write down the equation of the graph after this transformation.

[1 mark]

Answer $y = 2 - x^2$

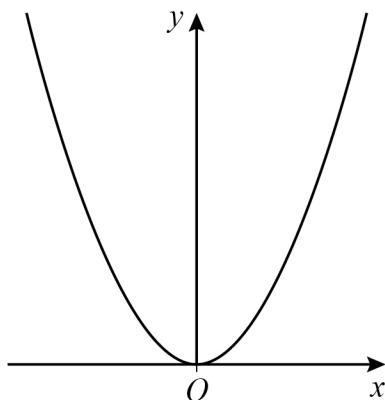
26 (c) $y = x^2$ is now transformed to give $y = (x - 2)^2$

Describe fully this single transformation.

[2 marks]

Translation by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

26 Here is a sketch of $y = x^2$



26 (a) The minimum point of $y = x^2$ is at $(0, 0)$

Write down the coordinates of the minimum point of $y = x^2 + 2$

[1 mark]

Answer (_____ , _____)

26 (b) The graph $y = x^2$ is reflected in the x axis.

Write down the equation of the graph after this transformation.

[1 mark]

Answer _____

26 (c) $y = x^2$ is now transformed to give $y = (x + 3)^2$

Describe fully this single transformation.

[2 marks]

END OF QUESTIONS