Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

GCSE MATHEMATICS
Example-Problem Past Paper

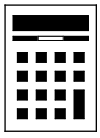
H

Higher Tier Paper 3 Calculator

June 2023

Materials

For this paper you must have:
- a calculator
- mathematical instruments.



Instructions
- Engage with the fully-worked solutions in full before attempting the shadow questions.
- Explain the fully-worked solutions to yourself, anticipating the next steps in the worked solutions, making links between the problems and the mathematics used to solve them.
- Apply the methods learnt from the fully-worked solutions to the shadow questions, writing down all workings in the spaces provided. Your thought process is important.
- Do all rough work in this book.

Information
- The marks for questions are shown in brackets.
- You may ask for more answer paper, graph paper and tracing paper.
- You should ask your teacher for help on a question if you don't understand a part of the fully-worked solution. Remember to be specific, understanding why the step was completed, rather than simply getting the correct answer.

Advice

In all calculations, show clearly how you work out your answer.

Where a calculator has been used, show clearly what you entered into the calculator.

Answer **all** questions in the spaces provided.

Do not write outside the box

1 The line with equation $y = 2x + 7$ intersects the y-axis at A.

Complete the coordinates of A.

[1 mark]

$y = mx + c$ (m: gradient, c: y-intercept)

Answer (0 , 7)

2 Write down a fraction equivalent to 1.875

[1 mark]

$\frac{1875}{1000}$

Answer $\frac{1875}{1000}$

3 Solve $5x + 11 = 3x + 19$

[2 marks]

$-3x \quad -3x$

$2x + 11 = 19$

$-11 \quad -11$

$2x = 8$

$:2 \quad :2$

$x = 4$

$x =$ 4

Answer **all** questions in the spaces provided.

Do not write outside the box

1 The line with equation $y = 6x - 3$ intersects the y-axis at A.

Complete the coordinates of A.

[1 mark]

Answer (0 , _____)

2 Write down a fraction equivalent to 2.925

[1 mark]

Answer _____

3 Solve $10x - 17 = 4x + 13$

[2 marks]

$x =$ _____

4

Turn over ►

Do not write outside the box

4 A map has a scale of 1 : 5000

How many **metres** are represented by a length of 4.5 cm on the map?

[2 marks]

1cm represents 5000cm = 50m

×50 (1m / 50m = 100cm / 5000cm) ×50

×4.5 (1cm / 4.5cm = 50m / 225m) ×4.5

Answer 225 m

5 The number of hedgehogs in England is expected to **reduce** by 4% each year.

Assume there are now 1 000 000 hedgehogs in England.

Work out the expected number of hedgehogs in England after **five** years.

You **must** show your working.

[3 marks]

$1\,000\,000 \times 0.96^5 = 815,372.6976$

Answer 815,373

Do not write outside the box

4 A map has a scale of 1 : 6500

How many **metres** are represented by a length of 3.8 cm on the map?

[2 marks]

Answer _____ m

5 The number of foxes in England is expected to **increase** by 2% each year.
Assume there are now 357 000 foxes in England.

Work out the expected number of foxes in England after **six** years.
You **must** show your working.

[3 marks]

Answer _____

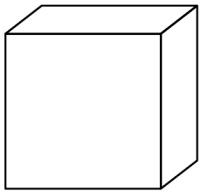
5

Turn over ►

Do not write outside the box

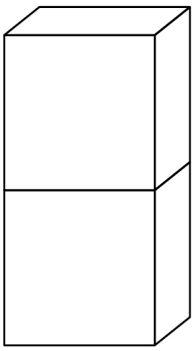
6 Here is cuboid A.

A



Cuboid B is made from **two** of cuboid A.

B



volume of A : volume of B = 1 : 2

Matthew says,

"surface area of A : surface area of B must be 1 : 2 because B is made of 2 of A."

Is Matthew correct?

Tick **one** box.

☐ Yes ☑ No ☐ Cannot tell

Give a reason for your answer.

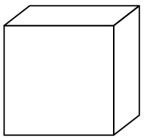
[2 marks]

Cuboid A is formed of 6 faces, whereas cuboid B is formed of 10 of these faces, without the two which are joined.

Do not write outside the box

6 Here is a cube A.

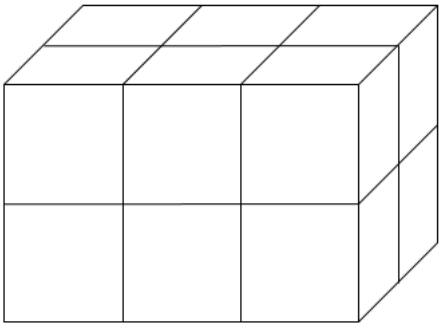
A



Not drawn accurately

Cuboid B is made from **twelve** of cube A.

B



volume of A : volume of B = 1 : 12

Henry says,
"surface area of A : surface area of B must be 1 : 12 because cuboid B is made of 12 of A."

Is Henry correct?

Tick **one** box.

☐ Yes ☐ No ☐ Cannot tell

Give a reason for your answer.

[2 marks]

2

Do not write outside the box

7 (a) Complete the table of values for $y = x^2 + 2x$

[2 marks]

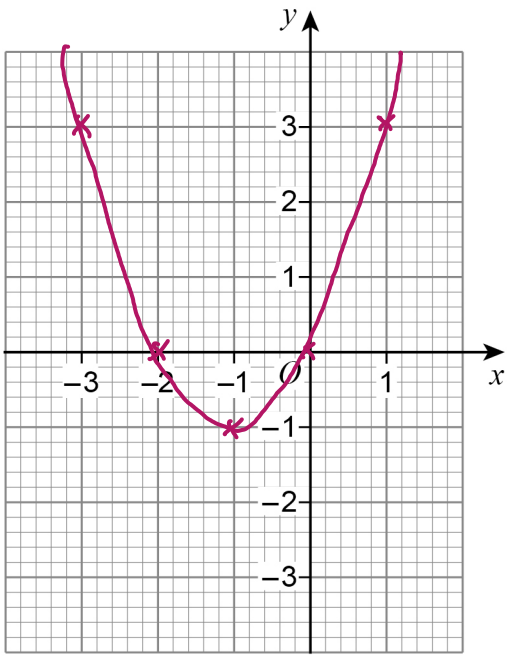
x	–3	–2	–1	0	1
y	3	0	–1	0	3

$(-2)^2 + 2(-2)$
$= 4 + (-4) = 0$

$(1)^2 + 2(1)$
$= 1 + 2 = 3$

7 (b) Draw the graph of $y = x^2 + 2x$ for values of x from –3 to 1

[2 marks]



Do not write outside the box

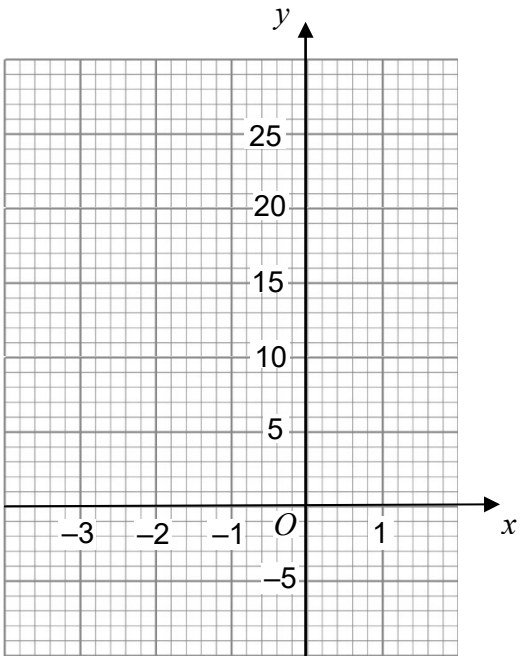
7 (a) Complete the table of values for $y = x^2 - 4x$

[2 marks]

x	–3	–2	–1	0	1
y	21		5	0	

7 (b) Draw the graph of $y = x^2 - 4x$ for values of x from –3 to 1

[2 marks]



4

Turn over ►

Do not write outside the box

8 Jing has £2450

She saves some and gives the rest to her four brothers.

money saved : money given to brothers = 2 : 5

She gives each of her **four** brothers the **same** amount.

Does each brother receive more than £430 ?

You **must** show your working.

[4 marks]

Saved O O 350 × 2 : 700

Given O O O O O 350 × 5 : 1750

2450 : 7 : 350

1750 : 4 : 437.5

Yes.

Do not write outside the box

8 Shirley has £5625

She saves some and donates the rest to charity.

money saved : money given to charity = 2 : 7

She gives each of **five** charities the **same** amount.

Does each charity receive more than £870 ?

You **must** show your working.

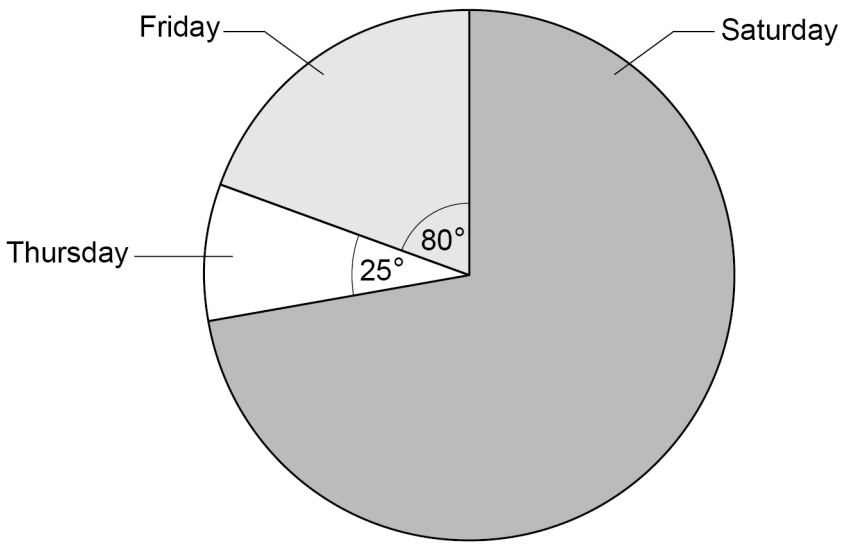
[4 marks]

4

Do not write outside the box

9 The pie chart shows information about people at a fair during three days.

Not drawn accurately



There were 132 **more** people on Friday than on Thursday.

Work out the number of people on Saturday.

[3 marks]

80° - 25° : 55°

:11 (55° | 132 people) :11

5° | 12

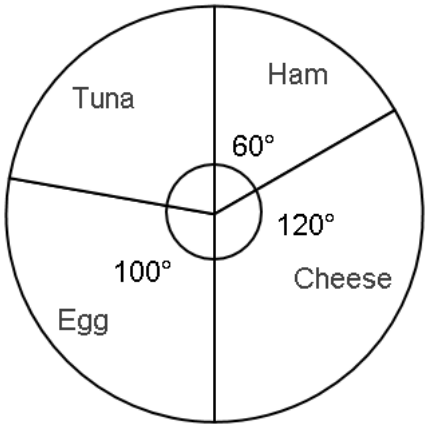
×51 (255° | 612) ×51

360° - (80° + 25°) : 255°

Answer 612

Do not write outside the box

9 The pie chart shows information about customers choice of sandwich filling.



Not drawn accurately

12 **more** customers chose egg than chose ham.

Work out the number of customers who chose tuna.

[3 marks]

Answer _____

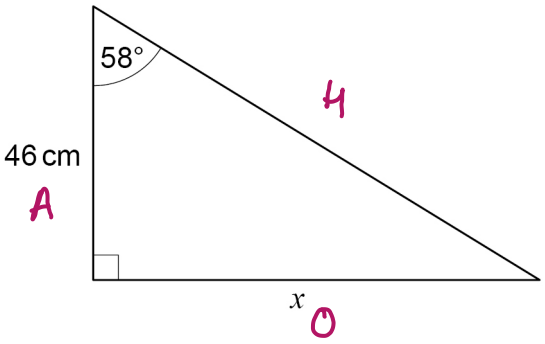
3

Turn over ►

Do not write outside the box

10 Use trigonometry to work out the value of x.

Not drawn accurately



H

A

O

[3 marks]

$\tan\theta = \frac{O}{A}$ $\tan(58°) = \frac{x}{46}$

$\times 46$ $\times 46$

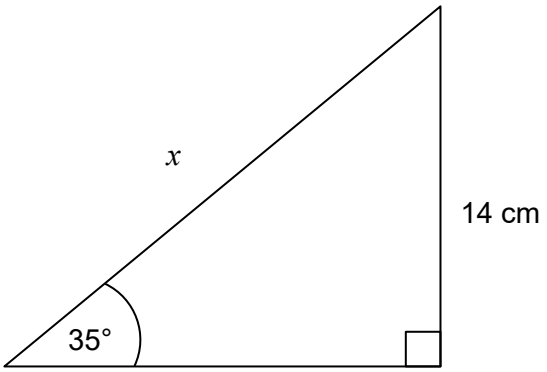
$\tan(58) \times 46 = x = 73.615388336$

$x =$ 73.62 cm

Do not write outside the box

10 Use trigonometry to work out the value of x.

Not drawn accurately



[3 marks]

$x =$ _____ cm

3

Do not write outside the box

11 Millie is estimating the value of $\dfrac{1}{\left(\sqrt[3]{8.34}\right)^2 \times 10.21}$

She rounds each decimal number to 1 significant figure.

11 (a) Work out Millie's estimate.

You **must** show your working.

[2 marks]

$\dfrac{1}{(\sqrt[3]{8})^2 \times 10} : \dfrac{1}{2^2 \times 10} : \dfrac{1}{40}$

Answer $\dfrac{1}{40}$

11 (b) Millie says,

"My estimate must be more than the exact value."

Without working out the exact value, give a reason how she can know this.

[1 mark]

Because each value was rounded down to estimate, the exact value is a division by a larger denominator, leading to a smaller value.

Do not write outside the box

11 Aiza is estimating the value of $\dfrac{2}{\left(\sqrt{4.36}\right)^3 \times 5.49}$

She rounds each decimal number to 1 significant figure.

11 (a) Work out Aiza's estimate.

You **must** show your working.

[2 marks]

Answer _____

11 (b) Aiza says,

"My estimate must be larger than the exact value."

Without working out the exact value, give a reason how she can know this.

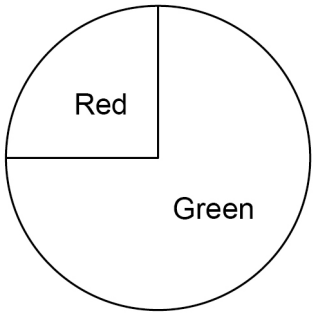
[1 mark]

3

Turn over ►

Do not write outside the box

12 Here is a **biased** spinner.



12 (a) Ali, Ben and Cary want to know the probability of spinning red on the biased spinner.

They each spin it and count how many times it lands on red and divide by the total number of spins.

Ali says — I spun red the most times

Ben says — I spun the spinner the most times

Cary says — My relative frequency of red is 0.25

Who had the best estimate for the probability of spinning red?

Give a reason for your answer.

[1 mark]

Ben because he conducted the most trials.

Do not write outside the box

12 (b) Dev spins the spinner 80 times.

He says,

"My relative frequency of red is 0.185"

Give a reason why his relative frequency must be wrong.

[1 mark]

80 × 0.185 : 14.8

Dev cannot have spun 'red' 14.8 times - this should be a whole number.

12 (c) Elena spins the spinner 125 times.

The relative frequency of red is 0.32

Work out how many times the spinner landed on **green**.

[2 marks]

Relative frequency of green : 1 - 0.32 : 0.68

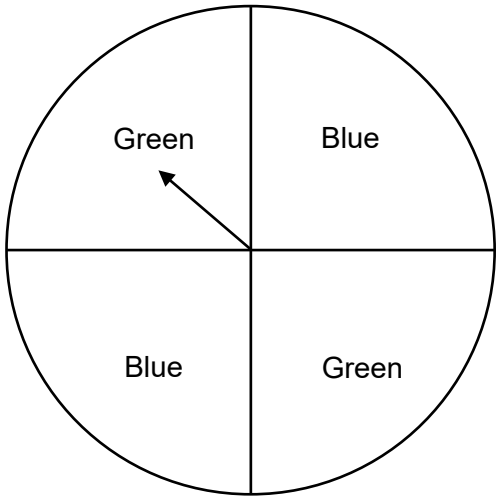
0.68 × 125 : 85

Answer 85

Turn over ►

Do not write outside the box

12 Here is a **biased** spinner.



12 (a) Ann, Bill and Celine want to know the probability of spinning blue on the biased spinner.

They each spin it and count how many times it lands on blue and divide by the total number of spins.

Ann says — I spun blue the most times

Bill says — I spun the spinner the most times

Celine says — My relative frequency of blue is 0.50

Who had the best estimate for the probability of spinning blue?

Give a reason for your answer.

[1 mark]

Do not write outside the box

12 (b) David spins the spinner 100 times.

He says,

"My relative frequency of blue is $\frac{1}{3}$"

Give a reason why his relative frequency must be wrong.

[1 mark]

12 (c) Emily spins the spinner 175 times.

The relative frequency of blue is 0.64

Work out how many times the spinner landed on **green**.

[2 marks]

Answer _____

4

Turn over ►

Do not write outside the box

13 Charlie is driving 293 miles home.

He

- leaves at 9.00 am
- travels the first 176 miles at an average speed of 48 mph
- drives the rest of the way at an average speed of 65 mph

Will he be home by 2.30 pm?

You **must** show your working.

[4 marks]

176 ÷ 3.6̇ = 3hrs 40 minutes
48 $(3\frac{2}{3})$

293 - 176 = 117

117 ÷ 1.8 = 1hr 48 minutes
65 $(1\frac{4}{5})$

3hrs 40 mins + 1hr 48 mins = 4hrs 88 mins = 5h 28 mins

9am + 5h 28m = 14:28 = 2.28pm

Yes.

Do not write outside the box

13 Daniel is driving 154 miles to visit his aunt.

He:

- leaves at 8.15 am
- travels the first 90 miles at an average speed of 50 mph
- drives the rest of the way at an average speed of 47 mph.

Will he be at his aunt's by 11.30 am?

You **must** show your working.

[4 marks]

4

Do not write outside the box

14 Kiran paid Income Tax and National Insurance on her annual salary.

Income Tax

0% of the first £12 570 of her annual salary

20% of the rest of her annual salary

National Insurance

0% of the first £9880 of her annual salary

13.25% of the rest of her annual salary

Kiran paid £5186 Income Tax.

How much National Insurance did she pay?

[4 marks]

×5 { £5186 | 20% / £25,930 | 100% } ×5 Taxable salary

25 930 + 12 570 : £38,500 salary

38500 - 9880 = £28,260

28260 × 0.1325 = £3,792.15

Answer £ 3,792.15

Do not write outside the box

14 Stephanie paid Income Tax and National Insurance on her annual salary.

Income Tax

0% of the first £14 700 of her annual salary

20% of the rest of her annual salary

National Insurance

0% of the first £6500 of her annual salary

15.75% of the rest of her annual salary

Stephanie paid £600 Income Tax.

How much National Insurance did she pay?

[4 marks]

Answer £ _____

4

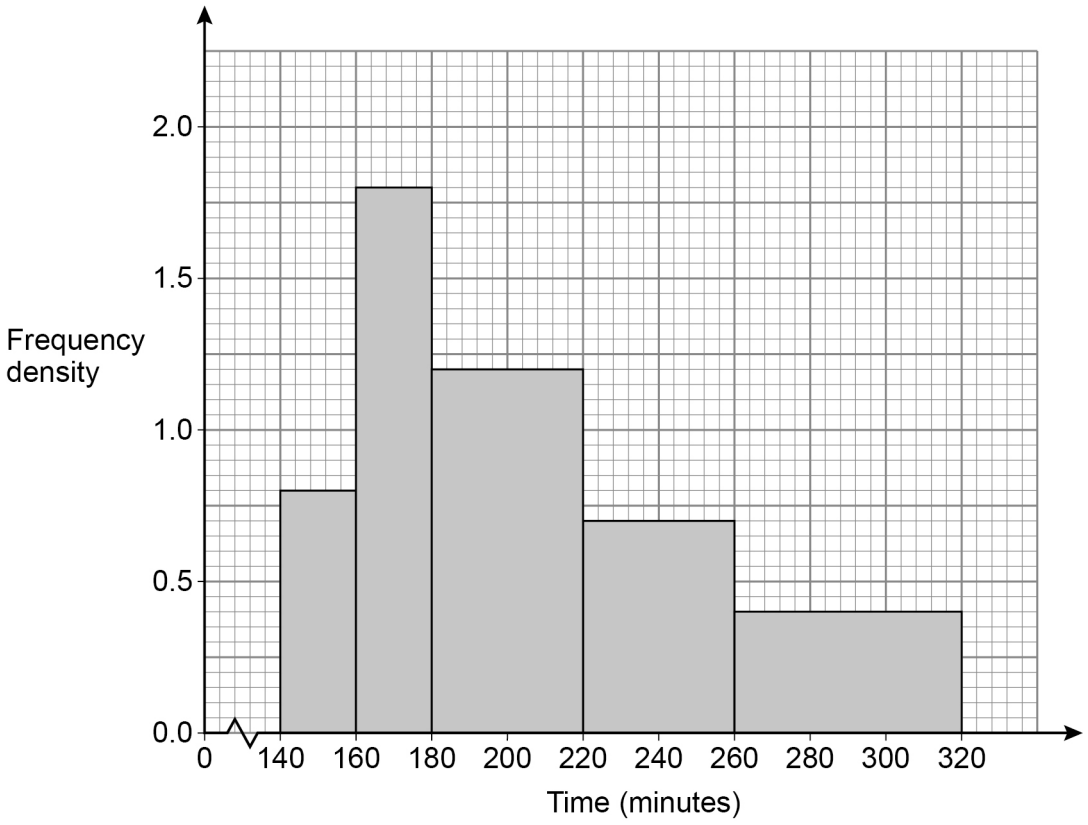
Turn over ►

Do not write outside the box

15 180 runners **started** a marathon.

Some of the runners did not complete it.

15 (a) The histogram represents the times of the runners who did complete the marathon.



How many runners did **not** complete the marathon?

[3 marks]

20 × 0.8 : 16

20 × 1.8 : 36

40 × 1.2 : 48

40 × 0.7 : 28

60 × 0.4 : 24

142 finishers 180 - 142 = 38

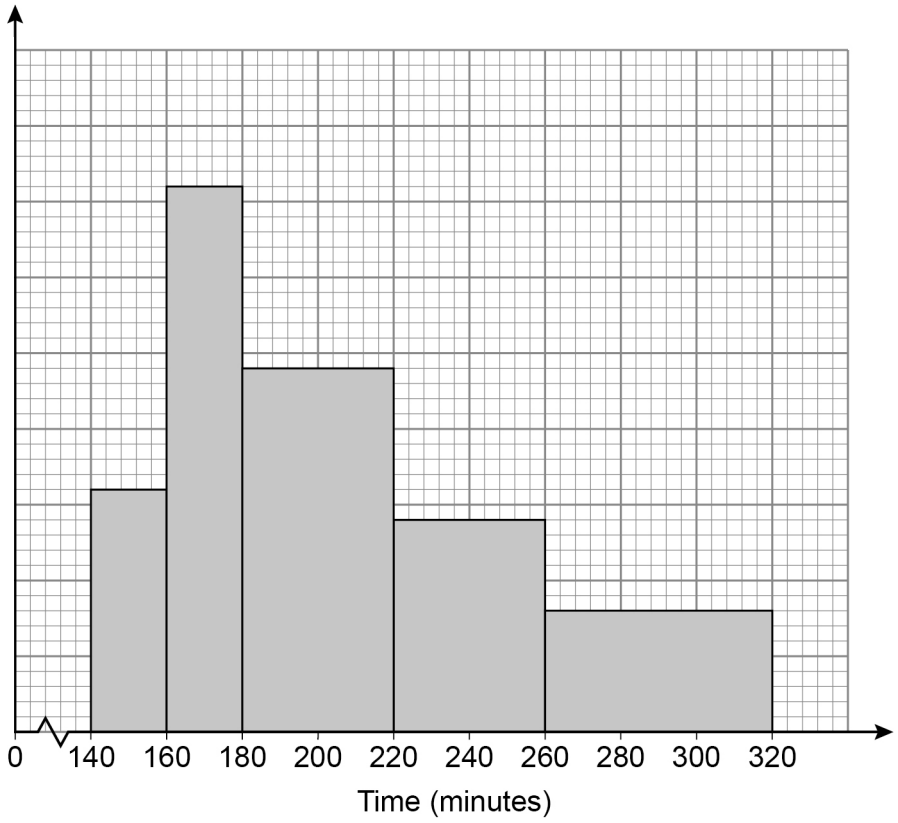
Answer 38

Do not write outside the box

15 Some runners from Coventry Running Club entered a marathon.

15 (a) The histogram represents the times of the runners from the club who completed the marathon.

48 runners finished the marathon between 260 and 320 minutes.



How many runners finished in under 220 minutes?

[3 marks]

Answer _____

3

Do not write outside the box

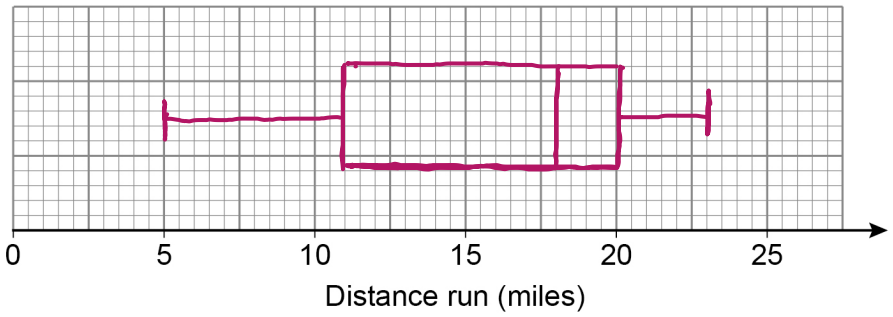
15 (b) The table shows information about the runners who did **not** complete the marathon.

	Distance run (miles)
Least distance	5
Greatest distance	23
Lower quartile	11
Median	18
Interquartile range	9

Upper quartile
= LQ + IQR
= 11 + 9 = 20

Draw a box plot to represent the information.

[3 marks]



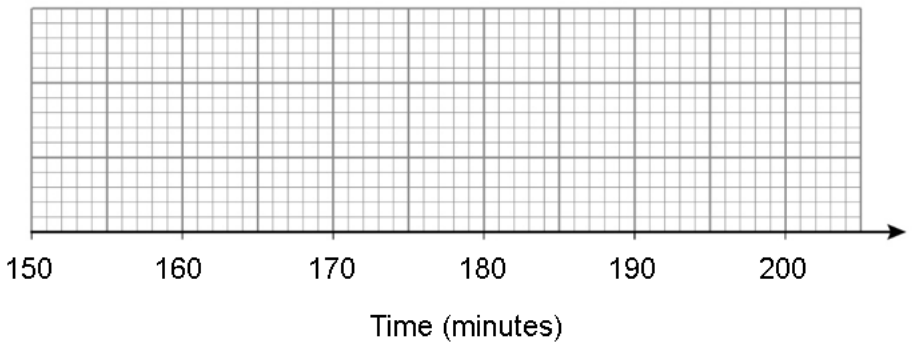
Do not write outside the box

15 (b) The table shows information about the runners who completed the marathon from Leicester Running Club.

	Time (minutes)
Least time	150
Greatest time	200
Lower quartile	163
Median	172
Interquartile range	24

Draw a box plot to represent the information.

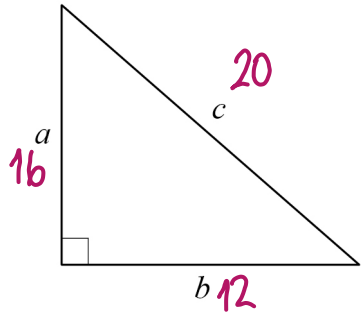
[3 marks]



3

Do not write outside the box

16



Not drawn accurately

In this right-angled triangle,

$a = 16$ cm

$a : c = 4 : 5$

Work out the area of the triangle.

[4 marks]

$\times 4 \begin{pmatrix} 4:5 \\ 16:20 \end{pmatrix} \times 5$

$20^2 - 16^2 = 400 - 256 = 144$

$b^2 = 144$

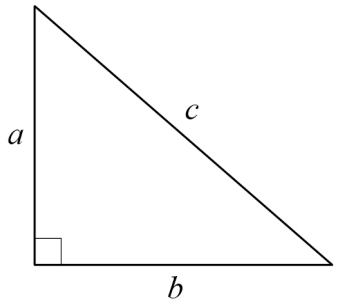
$b = \sqrt{144} = 12$

$\text{Area} = \frac{bh}{2} = \frac{16 \times 12}{2} = 96$

Answer 96 cm^2

Do not write outside the box

16



Not drawn accurately

In this right-angled triangle,

$c = 39$ cm

$c : a = 13 : 5$

Work out the area of the triangle.

[4 marks]

Answer _____ cm^2

4

Turn over ►

Do not write outside the box

17 Solve $\frac{x+8}{2} + \frac{9-x}{5} = 4$

[4 marks]

$\frac{x+8}{2} + \frac{9-x}{5} = \frac{5(x+8)}{10} + \frac{2(9-x)}{10}$

$= \frac{5x+40+18-2x}{10} = \frac{3x+58}{10}$

$\frac{3x+58}{10} = 4$

$\times 10 \quad \times 10$

$3x + 58 = 40$

$-58 \quad -58$

$3x = -18$

$\div 3 \quad \div 3$

$x = -6$

$x =$ -6

Do not write outside the box

17 Solve $\frac{x-4}{3}+\frac{10-x}{4}=1$

[4 marks]

$x =$ _____

4

Turn over ►

Do not write outside the box

18 $f(x) = x^2 + 6x$

$g(x) = 2x + 4$

18 (a) Show that $fg(x) = 4x^2 + 28x + 40$

[3 marks]

$fg(x) = f(g(x))$

$= g(x)^2 + 6(g(x))$

$= (2x+4)^2 + 6(2x+4)$

$= 4x^2 + 16x + 16 + 12x + 24$

$= 4x^2 + 28x + 40$

18 (b) Solve $fg(x) = -5$

[3 marks]

$4x^2 + 28x + 40 = -5$

$+5 \quad +5$

$4x^2 + 28x + 45 = 0$

$(2x+9)(2x+5) = 0$

$2x+9=0$ $\quad -9 \quad -9$

$2x = -9$ $\quad :2 \quad :2$

$x = -4.5$

$2x+5=0$ $\quad -5 \quad -5$

$2x = -5$ $\quad :2 \quad :2$

$x = -2.5$

Answer $x = -4.5, \quad x = -2.5$

Do not write outside the box

18 $f(x) = 3x^2 - x$

$g(x) = x + 3$

18 (a) Show that $fg(x) = 3x^2 + 17x + 24$

[3 marks]

18 (b) Solve $fg(x) = 5$

Give your answers correct to 2 decimal places.

[3 marks]

Answer _____

6

Do not write outside the box

19 Two integers have a difference of 6

The integers are multiplied together.

9 is then added.

Prove algebraically that the result is always a square number.

[3 marks]

x and $x+6$

$x(x+6) = x^2+6x$

x^2+6x+9

$x^2+6x+9 = (x+3)^2$

Do not write outside the box

19 Two integers have a difference of 2

The integers are multiplied together.

1 is then added.

Prove algebraically that the result is always a square number.

[3 marks]

3

Turn over ►

Do not write outside the box

20 (a) Sunil thinks that E and D are linked by the equation $E = \frac{36}{D}$

The graph shows the values of D and E for $2 \leqslant D \leqslant 6$

Choose **one** point on the graph and state if Sunil's equation is correct for that point.

[1 mark]

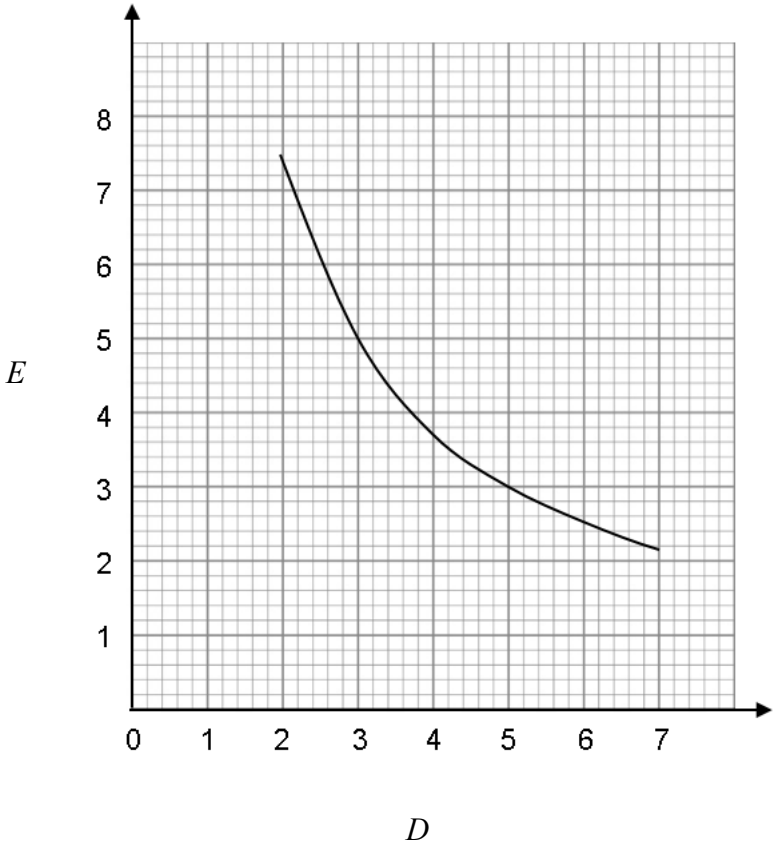
(5, 7) D = 5, E = 7 $\frac{36}{5} = 7.2$

Sunil's equation is not correct.

Do not write outside the box

20 (a) Sunil thinks that E and D are linked by the equation $E = \frac{14}{D}$

The graph shows the values of D and E for $2 \leqslant D \leqslant 7$



Choose **one** point on the graph and state if Sunil's equation is correct for that point.

[1 mark]

1

Do not write outside the box

20 (b) G is directly proportional to the square root of H.

$G : H = 3 : 2$ when $H = 16$

Work out $G : H$ when $H = 100$

[4 marks]

$G \propto \sqrt{H}$

$G = k\sqrt{H}$

$\times 8 \{ 3 : 2 \} \times 8$

$24 : 16$

$24 = k \times \sqrt{16}$

$24 = k \times 4$

$k = 6$

$G = 6\sqrt{H}$

$H = 100$ $G = 6 \times \sqrt{100} = 6 \times 10 = 60$

$G : H$ $\div 20 \{ 60 : 100 \} \div 20$

$3 : 5$

Answer 3 : 5

Do not write outside the box

20 (b) G is directly proportional to the square of H.

$G : H = 5 : 1$ when $H = 10$

Work out $G : H$ when $H = 20$

[4 marks]

Answer _____ : _____

4

Turn over ►

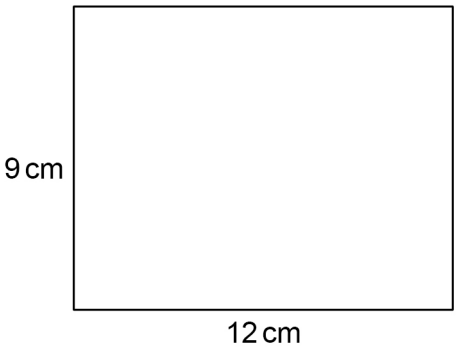
Do not write outside the box

21 A solid shape is made from centimetre cubes.

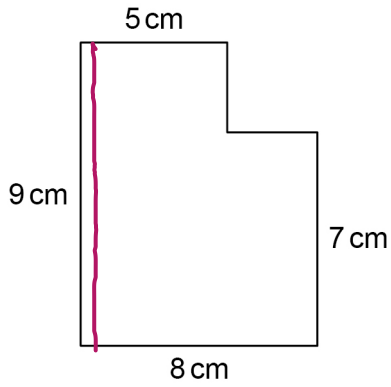
The front elevation and side elevation of the shape are shown.

Not drawn accurately

Front elevation



Side elevation



Work out

the **maximum** possible number of cubes in the shape

and

the **minimum** possible number of cubes in the shape.

[3 marks]

Maximum : Area of side x length

: (8 x 9 - 2 x 3) x 12

: (72 - 6) x 12 = 792

Minimum : 1 cube thick front + 1 cube thick side

: (9 x 12) + (7 x 9 - 2 x 3)

: 108 + 57

: 165

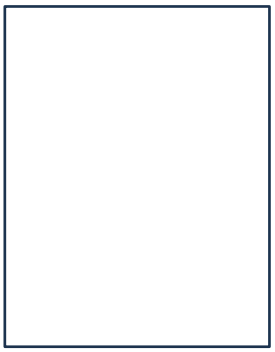
Maximum 792 Minimum 165

Do not write outside the box

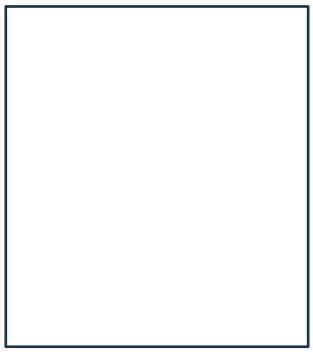
21 A solid shape is made from centimetre cubes.

The front elevation and side elevation of the shape are shown.

Front Elevation



Side elevation



Not drawn accurately

Work out

the **maximum** possible number of cubes in the shape

and

the **minimum** possible number of cubes in the shape.

[3 marks]

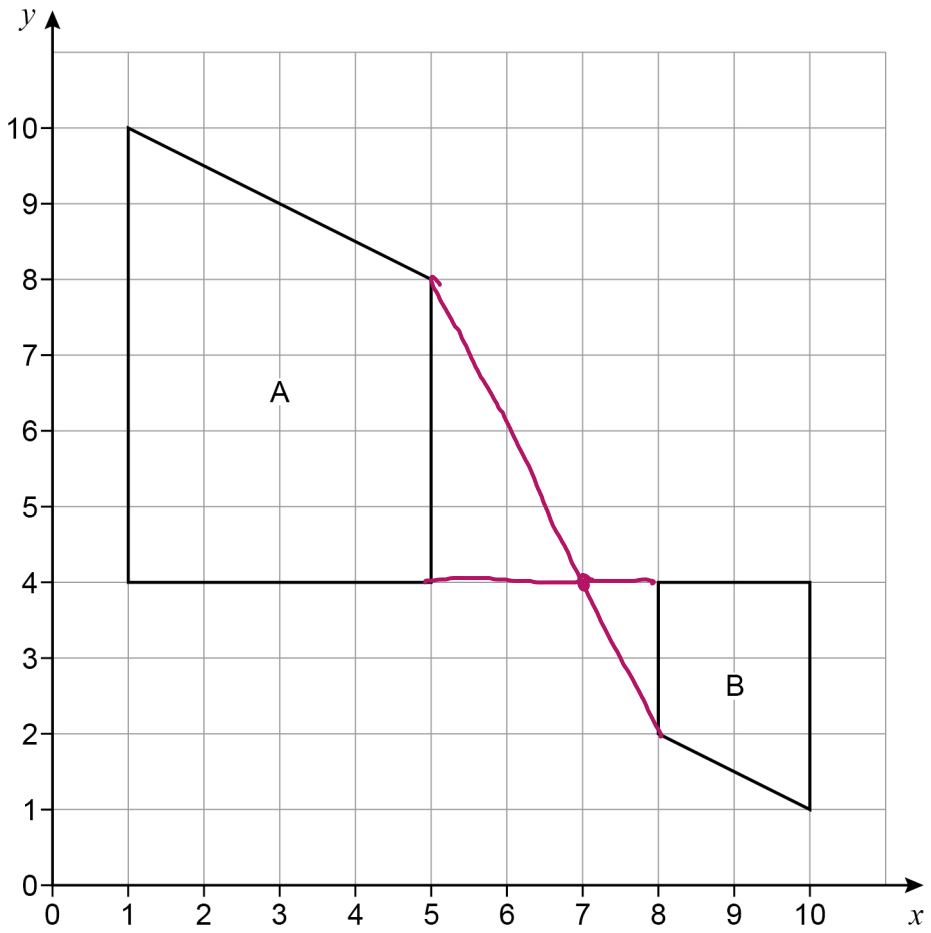
Maximum _____ Minimum _____

3

Turn over ►

Do not write outside the box

22 Shape A and shape B are shown on the grid.



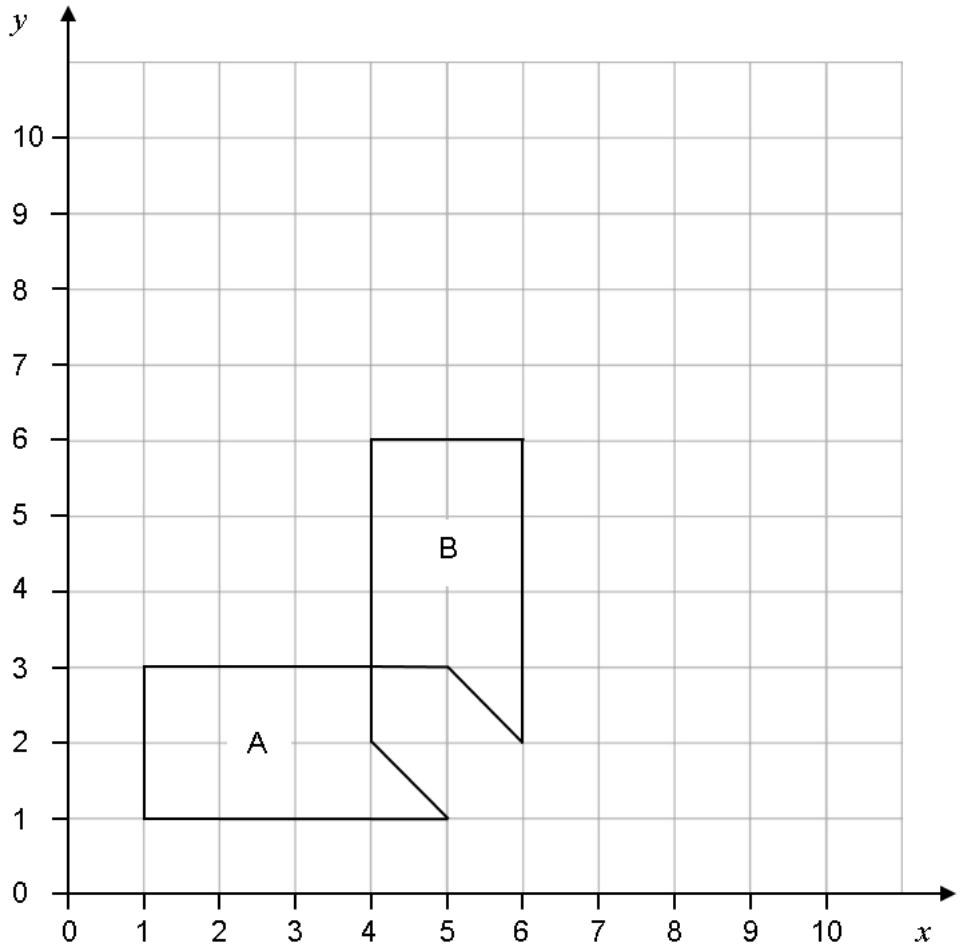
Describe the **single** transformation that maps shape A to shape B.

[3 marks]

Enlargement
Scale factor $-\frac{1}{2}$
Centre (7, 4)

Do not write outside the box

22 Shape A and shape B are shown on the grid.



Describe the **single** transformation that maps shape A to shape B.

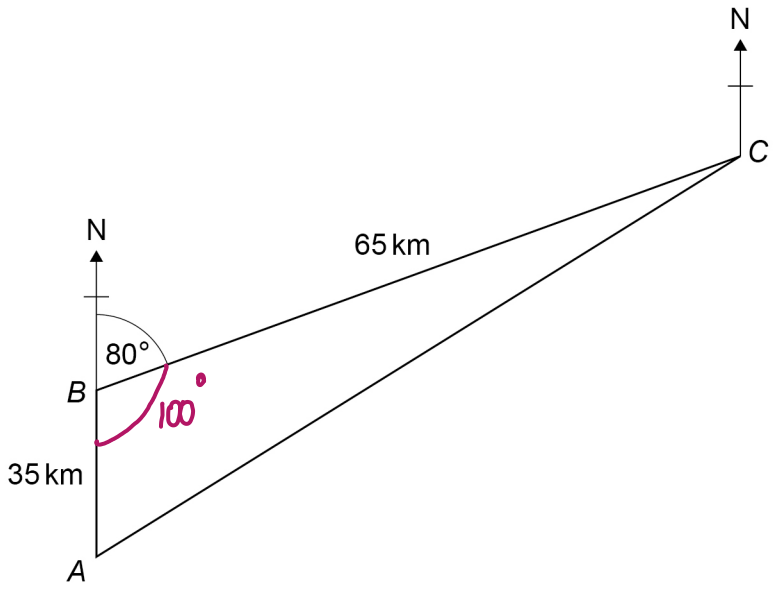
[3 marks]

3

Turn over ►

Do not write outside the box

23



A boat sails 35 km North from *A* to *B*.

From *B* the boat sails to *C* and then back to *A*.

23 (a) Show that the distance the boat sails from *C* to *A* is 79 km to the nearest km

You **must** show your working.

[2 marks]

$C^2 = a^2 + b^2 - 2ab\cos C$

$AC^2 = 35^2 + 65^2 - 2 \times 35 \times 65 \times \cos 100$

$= 6420.099208385$

$AC = \sqrt{6420.099208385} = 78.99429858\,2\text{km}$

$= 79\text{km}$

Do not write outside the box

23 (b) Work out the bearing of *A* from *C*.

[4 marks]

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \frac{35^2 + 79^2 - 65^2}{2 \times 35 \times 79}$

$A = \cos^{-1}\left(\frac{35^2 + 79^2 - 65^2}{2 \times 35 \times 79}\right) = 54.120962679$

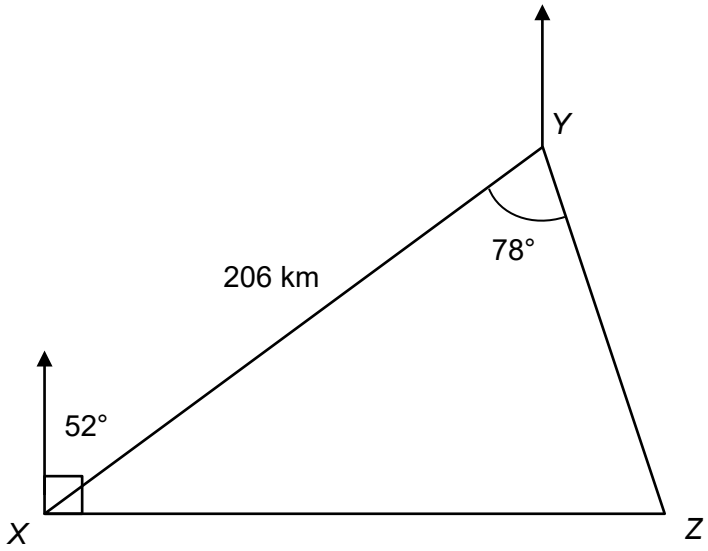
Answer 54.1 °

Turn over ►

Do not write outside the box

23

Not drawn accurately



A plane flies 206 km on a bearing of 052° from *X* to *Y*.

From *Y* the plane flies to *Z,* which is due East of X, and then back to *X*.

23 (a) Show that the distance the plane flies from *Y* to *Z* is 141 km to the nearest km.

You **must** show your working.

[2 marks]

Do not write outside the box

23 (b) Two boats leave the same port at the same time.

Boat A sails on a bearing of 157° at a speed of 16 mph.

Boat B sails on a bearing of 240° at a speed of 18 mph.

Calculate the bearing of boat A from boat B 90 minutes after they leave the port.

You may assume both boats are travelling at a constant speed.

[4 marks]

Answer _____ °

END OF QUESTIONS

6